Straight Lines and Pair of Straight Lines

Question1

The portion of the line 4x + 5y = 20 in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is :

[27-Jan-2024 Shift 1]

Options:

A.

8/5

B.

25/41

C.

2/5

D.

30/41

Answer: D

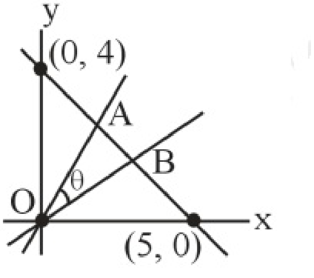
Solution:

Co-ordinates of A = $\left(\frac{5}{3}, \frac{8}{3}\right)$

Co-ordinates of B = $\left(\frac{10}{3}, \frac{4}{3}\right)$

Slope of OA = $m_1 = \frac{8}{5}$

Slope of OB = $m_2 = \frac{2}{5}$



$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \frac{\frac{6}{5}}{1 + \frac{16}{25}} = \frac{30}{41}$$

$$\tan\theta = \frac{30}{41}$$

Question2

Let R be the interior region between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin. The set of all values of a, for which the points $(a^2, a + 1)$ lie in R, is :

[27-Jan-2024 Shift 2]

Options:

A.

$(-3, -1) \cup \left(-\frac{1}{3}, 1\right)$

B.

$(-3, 0) \cup \left(\frac{1}{3}, 1\right)$

C.

$(-3, 0) \cup \left(\frac{2}{3}, 1\right)$

D.

$(-3, -1) \cup \left(\frac{1}{3}, 1\right)$

Answer: B

Solution:

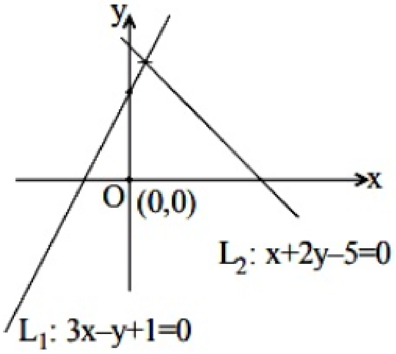
$P(a^2, a+1)$

$L_1 = 3x - y + 1 = 0$

Origin and P lies same side w.r.t. L_1

$\Rightarrow L_1(0) \cdot L_1(P) > 0$

$\therefore 3(a^2) - (a+1) + 1 > 0$



$2L_1 : 3x - y + 1 = 0$

$\Rightarrow 3a^2 - a > 0$

$a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$ .... (1)

Let $L_2 : x + 2y - 5 = 0$

Origin and P lies same side w.r.t. L_2

$\Rightarrow L_2(0) \cdot L_2(P) > 0$

$\Rightarrow a^2 + 2(a+1) - 5 < 0$

$\Rightarrow a^2 + 2a - 3 < 0$

$\Rightarrow (a+3)(a-1) < 0$

$\therefore a \in (-3, 1)$ .... (2)

Intersection of (1) and (2)

$a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$

Question3

Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point P(m,n) from the point Q(−2, −3) is

[27-Jan-2024 Shift 2]

Options:

A.

10

B.

6

C.

4

D.

8

Answer: A

Solution:

$2^m - 2^n = 56$

$2^n(2^{m-n} - 1) = 2^3 \times 7$

$2^n = 2^3$ and $2^{m-n} - 1 = 7$

$\Rightarrow n = 3$ and $2^{m-n} = 8$

$\Rightarrow n = 3$ and $m - n = 3$

$\Rightarrow n = 3$ and $m = 6$

$P(6, 3)$ and $Q(-2, -3)$

$PQ = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$

Hence option (1) is correct

Question4

If the sum of squares of all real values of α, for which the lines $2x - y + 3 = 0$, $6x + 3y + 1 = 0$ and $\alpha x + 2y - 2 = 0$ do not form a triangle is p, then the greatest integer less than or equal to p is ....

[27-Jan-2024 Shift 2]

Answer: 32

Solution:

$2x - y + 3 = 0$

$6x + 3y + 1 = 0$

$\alpha x + 2y - 2 = 0$

Will not form a Δ if $\alpha x+2y-2=0$ is concurrent with $2x-y+3=0$ and $6x+3y+1=0$ or parallel to either of them so

Case-1: Concurrent lines

$$\begin{vmatrix} 2 & -1 & 3 \\ 6 & 3 & 1 \\ \alpha & 2 & -2 \end{vmatrix}=0 \Rightarrow \alpha=\frac{4}{5}$$

Case-2 : Parallel lines

$$-\frac{\alpha}{2}=\frac{-6}{3} \text{ or } -\frac{\alpha}{2}=2$$

$$\Rightarrow \alpha=4 \text{ or } \alpha=-4$$

$$P=16+16+\frac{16}{25}$$

$$[P]=\left[32+\frac{16}{25}\right]=32$$

Question5

In a △ABC, suppose y = x is the equation of the bisector of the angle B and the equation of the side AC is 2x − y = 2. If 2AB = BC and the point A and B are respectively (4,6) and (α,β), then α + 2β is equal to

[29-Jan-2024 Shift 1]

Options:

A.

42

B.

39

C.

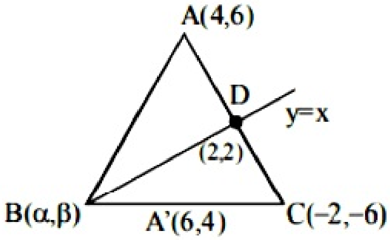
48

D.

45

Answer: A

Solution:



AD : DC = 1 : 2

$\frac{4-\alpha}{6-\alpha} = \frac{10}{8}$

$\alpha = \beta$

$\alpha = 14$ and $\beta = 14$

Question6

Let (5, a/4), be the circumcenter of a triangle with vertices A(a, −2), B(a, 6) and C (a/4, -2). Let α denote the circumradius, β denote the area and γ denote the perimeter of the triangle. Then α + β + γ is

[29-Jan-2024 Shift 1]

Options:

A.

60

B.

53

C.

62

D.

30

Answer: B

Solution:

$A(a,-2), B(a,6), C\left(\frac{a}{4},-2\right), O\left(5, \frac{a}{4}\right)$

$AO = BO$

$(a-5)^2 + \left(\frac{a}{4}+2\right)^2 = (a-5)^2 + \left(\frac{a}{4}-6\right)^2$

$a = 8$

$AB = 8, AC = 6, BC = 10$

$\alpha = 5, \beta = 24, \gamma = 24$

Question7

The distance of the point (2, 3) from the line $2x - 3y + 28 = 0$, measured parallel to the line $\sqrt{3}x - y + 1 = 0$, is equal to

[29-Jan-2024 Shift 2]

Options:

A.

$4\sqrt{2}$

B.

$6\sqrt{3}$

C.

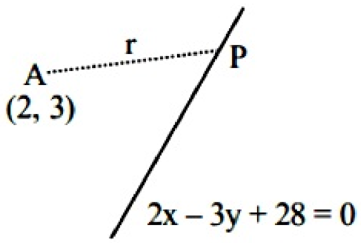
$3 + 4\sqrt{2}$

D.

$4 + 6\sqrt{3}$

Answer: D

Solution:



Writing P in terms of parametric co-ordinates $2 + r$

$\cos\theta, 3 + r\sin\theta$ as $\tan\theta = \sqrt{3}$

$P\left(2 + \frac{r}{2}, 3 + \frac{\sqrt{3}r}{2}\right)$

P must satisfy $2x - 3y + 28 = 0$

So, $2\left(2 + \frac{r}{2}\right) - 3\left(3 + \frac{\sqrt{3}r}{2}\right) + 28 = 0$

We find $r = 4 + 6\sqrt{3}$

Question8

Let A be the point of intersection of the lines $3x + 2y = 14$, $5x - y = 6$

and B be the point of intersection of the lines 4x + 3y = 8, 6x + y = 5. The distance of the point P(5, −2) from the line AB is

[29-Jan-2024 Shift 2]

Options:

A.

13/2

B.

8

C.

5/2

D.

6

Answer: D

Solution:

Solving lines L_1 (3x + 2y = 14) and L_2(5x − y = 6) to get A(2, 4) and solving lines L_3(4x + 3y = 8) and L_4(6x + y = 5) to get B (1/2, 2).

Finding Eqn. of AB : $4x - 3y + 4 = 0$

Calculate distance PM

$$\Rightarrow \left| \frac{4(5) - 3(-2) + 4}{5} \right| = 6$$

Question9

A line passing through the point A(9, 0) makes an angle of 30° with the positive direction of x-axis. If this line is rotated about A through an angle of 15° in the clockwise direction, then its equation in the new position is

[30-Jan-2024 Shift 1]

Options:

A.

$$\frac{y}{\sqrt{3}-2} + x = 9$$

B.

$$\frac{x}{\sqrt{3}-2}+y=9$$

C.

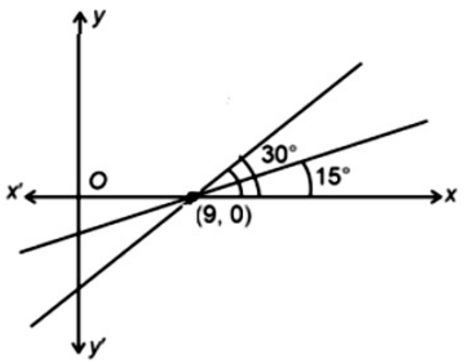
$$\frac{x}{\sqrt{3}+2}+y=9$$

D.

$$\frac{y}{\sqrt{3}+2}+x=9$$

Answer: A

Solution:



$Eq^{n}: y-0=\tan 15^{\circ}(x-9) \Rightarrow y=(2-\sqrt{3})(x-9)$

Question10

If $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$ is the locus of a point, which moves such that it is always equidistant from the lines $x + 2y + 7 = 0$ and $2x - y + 8 = 0$, then the value of $g + c + h - f$ equals

[30-Jan-2024 Shift 2]

Options:

A.

14

B.

6

C.

8

D.

29

Answer: A

Solution:

Cocus of point P(x, y) whose distance from

Gives

$x+2y+7=0$ & $2x-y+8=0$ are equal is

$$\frac{x+2y+7}{\sqrt{5}}=\pm\frac{2x-y+8}{\sqrt{5}}$$

$$(x+2y+7)^2-(2x-y+8)^2=0$$

Combined equation of lines

$$(x-3y+1)(3x+y+15)=0$$

$$3x^2-3y^2-8xy+18x-44y+15=0$$

$$x^2-y^2-\frac{8}{3}xy+6x-\frac{44}{3}y+5=0$$

$$x^2-y^2+2hxy+2gx2+2fy+c=0$$

$$h=\frac{4}{3}, g=3, f=-\frac{22}{3}, c=5$$

$$g+c+h-f=3+5-\frac{4}{3}+\frac{22}{3}=8+6=14$$

Question11

Let α, β, γ, δ ∈ Z and let A(α, β), B(1, 0), C(γ, δ) and D(1, 2) be the vertices of a parallelogram ABCD. If AB = √10 and the points A and C lie on the line 3y = 2x + 1, then 2(α + β + γ + δ) is equal to

[31-Jan-2024 Shift 1]

Options:

A.

10

B.

5

C.

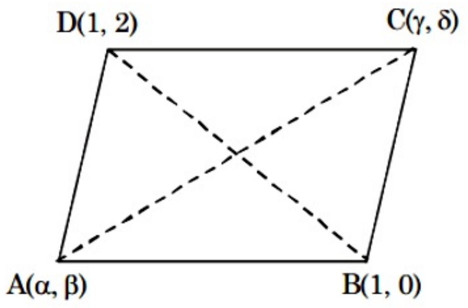
12

D.

8

Answer: D

Solution:



Let E is mid point of diagonals

$\frac{\alpha+\gamma}{2}=\frac{1+1}{2}$ & $\frac{\beta+\delta}{2}=\frac{2+0}{2}$

$\alpha+\gamma=2\ \beta+\delta=2$

$2(\alpha+\beta+\gamma+\delta)=2(2+2)=8$

Question12

Let A(a, b), B(3, 4) and (−6, −8) respectively denote the centroid, circumcentre and orthocentre of a triangle. Then, the distance of the point P(2a + 3, 7b + 5) from the line 2x + 3y − 4 = 0 measured parallel to the line x − 2y − 1 = 0 is

[31-Jan-2024 Shift 2]

Options:

A.

$\frac{15\sqrt{5}}{7}$

B.

$\frac{17\sqrt{5}}{6}$

C.

$\frac{17\sqrt{5}}{7}$

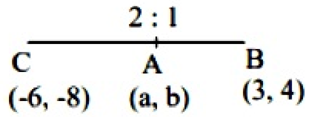
D.

$\frac{\sqrt{5}}{17}$

Answer: C

Solution:

$A(a, b)$, $B(3, 4)$, $C(-6, -8)$



$\Rightarrow a = 0, b = 0 \quad \Rightarrow P(3, 5)$

Distance from P measured along $x - 2y - 1 = 0$

$\Rightarrow x = 3 + r\cos\theta, \quad y = 5 + r\sin\theta$

Where $\tan\theta = \frac{1}{2}$

$r(2\cos\theta + 3\sin\theta) = -17$

$\Rightarrow r = \left|\frac{-17\sqrt{5}}{7}\right| = \frac{17\sqrt{5}}{7}$

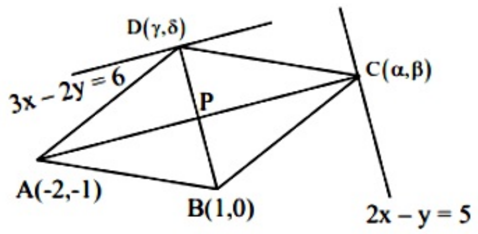
Question13

Let A(−2, −1), B(1, 0), C(α, β) and D(γ, δ) be the vertices of a parallelogram ABCD. If the point C lies on 2x − y = 5 and the point D lies on 3x − 2y = 6, then the value of |α + β + γ + δ| is equal to____

[31-Jan-2024 Shift 2]

Answer: 32

Solution:



$P \equiv \left(\frac{\alpha - 2}{2}, \frac{\beta - 1}{2}\right) \equiv \left(\frac{\gamma + 1}{2}, \frac{\delta}{2}\right)$

$\frac{\alpha - 2}{2} = \frac{\gamma + 1}{2}$ and $\frac{\beta - 1}{2} = \frac{\delta}{2}$

$\Rightarrow \alpha - \gamma = 3 \ldots..(1), \beta - \delta = 1 \ldots\ldots (2)$

Also, (γ, δ) lies on $3x - 2y = 6$

$3\gamma - 2\delta = 6$ ....(3)

and (α, β) lies on $2x - y = 5$

$\Rightarrow 2\alpha - \beta = 5$ . ...(4)

Solving (1), (2), (3), (4)

$\alpha = -3, \beta = -11, \gamma = -6, \delta = -12$

$|\alpha + \beta + \gamma + \delta| = 32$

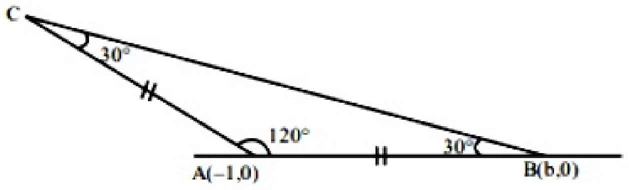
Question14

Let ABC be an isosceles triangle in which A is at (−1, 0), ∠A = 2π/3, AB = AC and B is on the positive x-axis. If BC = 4√3 and the line BC intersects the line y = x + 3 at (α, β), then β^4/α^2 is:

[1-Feb-2024 Shift 2]

Answer: 36

Solution:



$$\frac{c}{\sin 30^\circ} = \frac{4\sqrt{3}}{\sin 120^\circ} \text{ [By sine rule]}$$

$2c = 8 \Rightarrow c = 4$

$AB = |(b+1)| = 4$

$b = 3, m_{AB} = 0$

$m_{BC} = \frac{-1}{\sqrt{3}}$

BC : $-y = \frac{-1}{\sqrt{3}}(x-3)$

$\sqrt{3}y + x = 3$

Point of intersection : $y = x + 3, \sqrt{3}y + x = 3$

$(\sqrt{3}+1)y = 6$

$y = \frac{6}{\sqrt{3}+1}$

$x = \frac{6}{\sqrt{3}+1} - 3$

$= \frac{6 - 3\sqrt{3} - 3}{\sqrt{3}+1}$

$= 3\frac{(1-\sqrt{3})}{(1+\sqrt{3})} = \frac{-6}{(1+\sqrt{3})^2}$

$\frac{\beta^4}{\alpha^2} = 36$

Question15

Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area }(\triangle PQR)}{\text{Area }(\triangle ABC)}$ is equal to
[24-Jan-2023 Shift 1]

Options:

A. 4

B. 3

C. 2

D. −

Answer: B

Solution:

Let P is $\vec{0}$, Q is $\vec{q}$ and R is $\vec{r}$

A is $\frac{2\vec{q} + \vec{r}}{3}$, B is $\frac{2\vec{r}}{3}$ and C is $\frac{\vec{q}}{3}$

Area of $\triangle PQR$ is $= \frac{1}{2}\left|\vec{q} \times \vec{r}\right|$

Area of $\triangle ABC$ is $\frac{1}{2}\left|\vec{AB} \times \vec{AC}\right|$

$\vec{AB} = \vec{r} - 2\frac{\vec{q}}{3}, \vec{AC} = -\vec{r} - \frac{\vec{q}}{3}$

Area of $\triangle ABC = \frac{1}{6}\left|\vec{q} \times \vec{r}\right|$

$\frac{\text{Area }(\triangle PQR)}{\text{Area}(\triangle ABC)} = 3$

Question16

The equations of the sides AB and AC of a triangle ABC are $(\lambda + 1)x + \lambda y = 4$ and $\lambda x + (1 - \lambda)y + \lambda = 0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is [24-Jan-2023 Shift 2]

Options:

A. $\sqrt{6}$

B. $2\sqrt{2}$

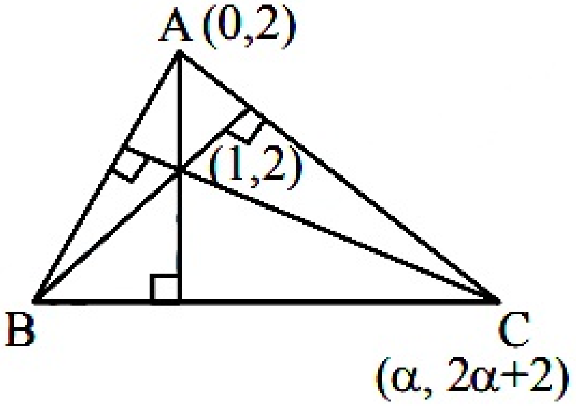
C. 2

D. 4

Answer: B

Solution:

AB : $(\lambda + 1)x + \lambda y = 4$
AC : $\lambda x + (1 - \lambda)y + \lambda = 0$
Vertex A is on y-axis
$\Rightarrow x = 0$



So $y = \frac{4}{\lambda}$, $y = \frac{\lambda}{\lambda - 1}$

$\Rightarrow \frac{4}{\lambda} = \frac{\lambda}{\lambda - 1}$

$\Rightarrow \lambda = 2$

AB : $3x + 2y = 4$

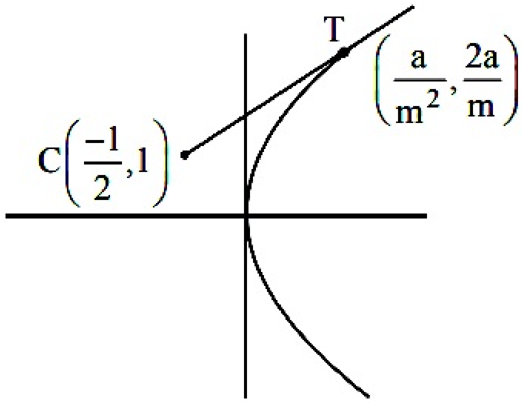
AC : $2x - y + 2 = 0$

$\Rightarrow A(0, 2)$ Let $C(\alpha, 2\alpha + 2)$

Now (Slope of Altitude through C) $\left(-\frac{3}{2}\right) = -1$

$\left(\frac{2\alpha}{\alpha-1}\right)\left(-\frac{3}{2}\right) = -1 \Rightarrow \alpha = -\frac{1}{2}$

So C $\left(-\frac{1}{2}, 1\right)$



Let Equation of tangent be $y = mx + \frac{3}{2m}$

$m^2 + 2m - 3 = 0$

$\Rightarrow m = 1, -3$

So tangent which touches in first quadrant at T is

$T \equiv \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$\equiv \left(\frac{3}{2}, 3\right)$

$\Rightarrow CT = \sqrt{4+4} = 2\sqrt{2}$

Question17

The equations of two sides of a variable triangle are x = 0 and y = 3, and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is:
[25-Jan-2023 Shift 2]

Options:

A. $4y^2 - 18y - 3x - 18 = 0$

B. $4y^2 + 18y + 3x + 18 = 0$

C. $4y^2 - 18y + 3x + 18 = 0$

D. $4y^2 - 18y - 3x + 18 = 0$

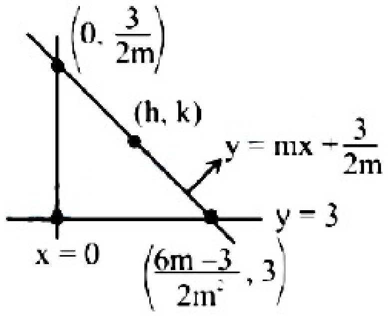
Answer: C

Solution:

Solution:

$y^2 = 6x \& y^2 = 4ax$

$\Rightarrow 4a = 6 \Rightarrow a = \frac{3}{2}$



$y = mx + \frac{3}{2m}$; $(m \neq 0)$

$h = \frac{6m-3}{4m^2}$, $k = \frac{6m+3}{4m}$, Now eliminating m and we get

$\Rightarrow 3h = 2(-2k^2 + 9k - 9)$

$\Rightarrow 4y^2 - 18y + 3x + 18 = 0$

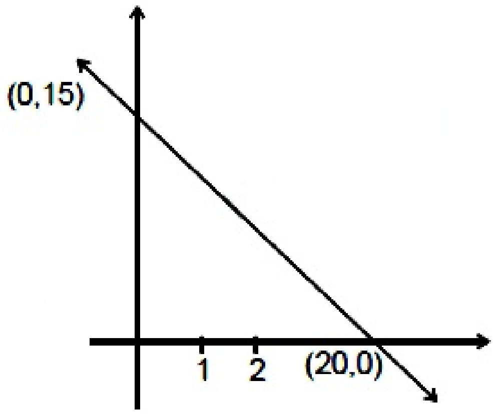
Question18

A triangle is formed by X - axis, Y - axis and the line 3x + 4y = 60. Then the number of points P(a, b)which lie strictly inside the triangle, where a is an integer and b is a multiple of a, is ________.
[25-Jan-2023 Shift 2]

Answer: 31

Solution:

If $x = 1$, $y = \frac{57}{4} = 14.25$



$(1, 1)(1, 2) - (1, 14) \Rightarrow 14$ pts.

If $x = 2$, $y = \frac{27}{2} = 13.5$

$(2, 2)(2, 4)...(2, 12) \Rightarrow 6$ pts.

If $x = 3$, $y = \frac{51}{4} = 12.75$

$(3, 3)(3, 6) - (3, 12) \Rightarrow 4$ pts.

If $x = 4$, $y = 12$

$(4, 4)(4, 8) \Rightarrow 2$ pts.

If $x = 5$. $y = \frac{45}{4} = 11.25$

$(5, 5), (5, 10) \Rightarrow 2$ pts.

If $x = 6$, $y = \frac{21}{2} = 10.5$

$(6, 6) \Rightarrow 1$ pt

If x = 7, y = $\frac{39}{4}$ = 9.75
(7, 7) $\Rightarrow$ 1 pt.
If x = 8, y = 9
(8, 8) $\Rightarrow$ 1 pt.
If x = 9y = $\frac{33}{4}$ = 8.25 $\Rightarrow$ no pt.
Total = 31 pts.

Question19

A light ray emits from the origin making an angle 30° with the positive x-axis. After getting reflected by the line x + y = 1, if this ray intersects x-axis at Q, then the abscissa of Q is [29-Jan-2023 Shift 1]

Options:

A. $\frac{2}{(\sqrt{3}-1)}$

B. $\frac{2}{3+\sqrt{3}}$

C. $\frac{2}{3-\sqrt{3}}$

D. $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$

Answer: B

Solution:

Solution:
Slope of reflected ray = $\tan 60^{\circ} = \sqrt{3}$
Line $y = \frac{x}{\sqrt{3}}$ intersect y + x = 1 at $\left(\frac{\sqrt{3}}{\sqrt{3}+1}, \frac{1}{\sqrt{3}+1}\right)$
Equation of reflected ray is
$y - \frac{1}{\sqrt{3}+1} = \sqrt{3}\left(x - \frac{\sqrt{3}}{\sqrt{3}+1}\right)$
Put y = 0 $\Rightarrow x = \frac{2}{3+\sqrt{3}}$

Question20

Let B and C be the two points on the line y + x = 0 such that B and C are symmetric with respect to the origin. Suppose A is a point on y − 2x = 2 such that △ABC is an equilateral triangle. Then, the area of the △ABC is [29-Jan-2023 Shift 1]

Options:

A. $3\sqrt{3}$

B. $2\sqrt{3}$

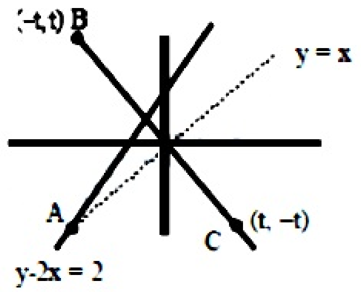
C. $\frac{8}{\sqrt{3}}$

D. $\frac{10}{\sqrt{3}}$

Answer: C

Solution:

Solution:



At A $x = y$
$Y - 2x = 2$
$(-2, -2)$
Height from line $x + y = 0$
$h = \frac{4}{\sqrt{2}}$
Area of Δ = $\frac{\sqrt{3}}{4}\frac{h^2}{\sin^2 60} = \frac{8}{\sqrt{3}}$

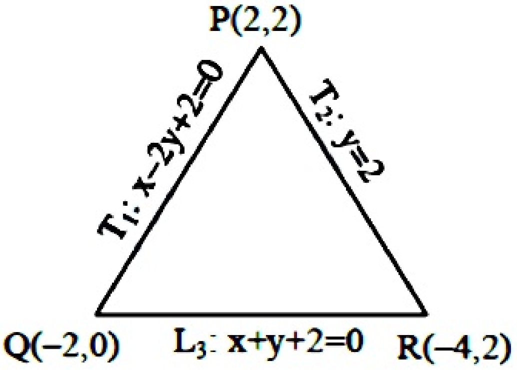
Question21

A triangle is formed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to _______.
[29-Jan-2023 Shift 2]

Answer: 10

Solution:

$S_1 : y^2 = 2x$ $S_2 : x^2 + y^2 = 4x$
$P(2, 2)$ is common point on S_1&S_2
T_1 is tangent to S_1 at P $\Rightarrow T_1 : y \cdot 2 = x + 2$
$\Rightarrow T_1 : x - 2y + 2 = 0$
T_2 is tangent to S_2 at P $\Rightarrow T_2 : x \cdot 2 + y \cdot 2 = 2(x + 2)$
$\Rightarrow T_2 : y = 2$
$L_3 : x + y + 2 = 0$ is third line



$PQ = a = \sqrt{20}$
$QR = b = \sqrt{8}$
$RP = c = 6$

$\text{Area } (\Delta PQR) = \Delta = \frac{1}{2} \times 6 \times 2 = 6$

$\therefore r = \frac{abc}{4\Delta} = \frac{\sqrt{160}}{4} = \sqrt{10} \Rightarrow r^2 = 10$

Question22

A straight line cuts off the intercepts OA = a and OB = b on the positive directions of x-axis and y− axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y-axis and the area of $\triangle OAB$ is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to:
[30-Jan-2023 Shift 1]

Options:

A. $\frac{392}{3}$

B. 196

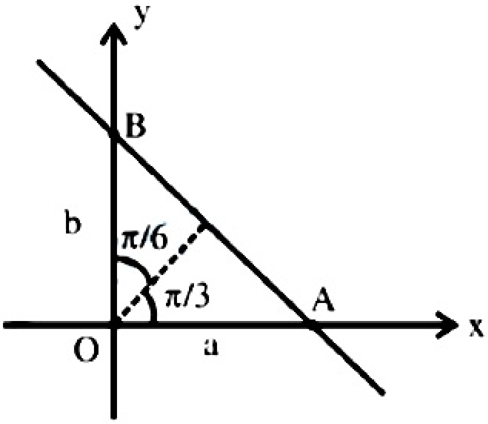
C. $\frac{196}{3}$

D. 98

Answer: A

Solution:

Solution:



Equation of straight line : $\frac{x}{a} + \frac{y}{b} = 1$

Or $x\cos\frac{\pi}{3} + y\sin\frac{\pi}{3} = p$

$\frac{x}{2} + \frac{y\sqrt{3}}{2} = p$

$\frac{x}{3p} + \frac{y}{2p} = 1$

Comparing both : $a = 2p$, $b = \frac{2p}{\sqrt{3}}$

Now area of $\triangle OAB = \frac{1}{2} \cdot ab = \frac{98}{3} \cdot \sqrt{3}$

$\frac{1}{2} \cdot 2p \cdot \frac{2p}{\sqrt{3}} = \frac{98}{3} \cdot \sqrt{3}$

$p^2 = 49$

$a^2 - b^2 = 4p^2 - \frac{4p^2}{3} = \frac{2}{3}4p^2$

$= \frac{8}{3} \cdot 49 = \frac{392}{3}$

Question23

If the orthocentre of the triangle, whose vertices are (1, 2), (2, 3) and (3, 1) is (α, β), then the quadratic equation whose roots are α + 4β and 4α + β, is [1-Feb-2023 Shift 1]

Options:

A. $x^2 - 19x + 90 = 0$

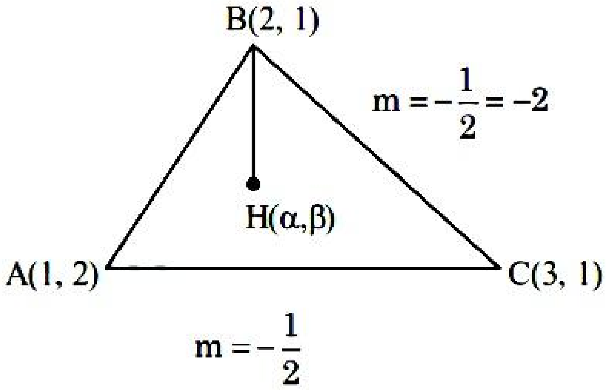
B. $x^2 - 18x + 80 = 0$

C. $x^2 - 22x + 120 = 0$

D. $x^2 - 20x + 99 = 0$

Answer: D

Solution:



Here $mBH \times mAC = -1$

$\left(\frac{\beta - 3}{\alpha - 2}\right)\left(\frac{1}{-2}\right) = -1$

$\beta - 3 = 2\alpha - 4$

$\beta = 2\alpha - 1$

$m_{AH} \times m_{BC} = -1$

$\Rightarrow \left(\frac{\beta - 2}{\alpha - 1}\right)(-2) = -1$

$\Rightarrow\ 2\beta - 4 = \alpha - 1$
$\Rightarrow\ 2(2\alpha - 1) = \alpha + 3$
$\Rightarrow\ 3\alpha = 5$

$\alpha = \frac{5}{3}, \beta = \frac{7}{3} \Rightarrow H\left(\frac{5}{3}, \frac{7}{3}\right)$

$\alpha + 4\beta = \frac{5}{3} + \frac{28}{3} = \frac{33}{3} = 11$

$\beta + 4\alpha = \frac{7}{3} + \frac{20}{3} = \frac{27}{3} = 9$

$x^2 - 20x + 99 = 0$

Question24

The combined equation of the two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ can be written as $(ax + by + c)(a'x + b'y + c') = 0$
The equation of the angle bisectors of the lines represented by the equation $2x^2 + xy - 3y^2 = 0$ is
[1-Feb-2023 Shift 1]

Options:

A. $3x^2 + 5xy + 2y^2 = 0$

B. $x^2 - y^2 + 10xy = 0$

C. $3x^2 + xy - 2y^2 = 0$

D. $x^2 - y^2 - 10xy = 0$

Answer: D

Solution:

Solution:
Equation of the pair of angle bisector for the homogenous equation $ax^2 + 2hxy + b^2 = 0$ is given as
$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$
Here $a = 2$, $h = 1 / 2$&$b = -3$
Equation will become
$\frac{x^2 - y^2}{2 - (-3)} = \frac{xy}{1 / 2}$
$x^2 - y^2 = 10xy$
$x^2 - y^2 - 10xy = 0$

Question25

The straight lines 1_1 and 1_2 pass through the origin and trisect the line segment of the line L : $9x + 5y = 45$ between the axes. If m_1 and m_2 are the slopes of the lines 1_1 and 1_2, then the point of intersection of the line $y = (m_1 + m_2)x$ with L lies on.
[6-Apr-2023 shift 1]

Options:

A. $6x + y = 10$

B. $6x - y = 15$

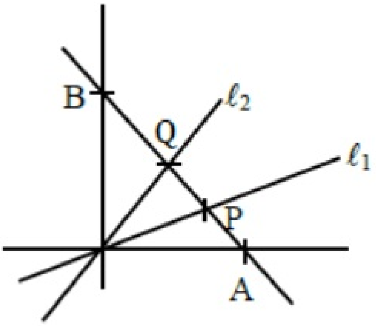
C. $y - 2x = 5$

D. $y - x = 5$

Answer: D

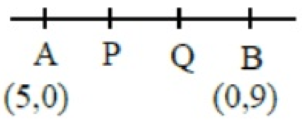
Solution:

Solution:



$L : 9x + 5y = 45$

$A(5,0)$, $B : (0,9)$



$\rightarrow P_x = \frac{2 \times 5 + 1 \times 0}{1 + 2} = \frac{10}{3}$

$P_y = \frac{0 \times 2 + 9 \times 1}{1 + 2} = 3$

$P : \left(\frac{10}{3}, 3\right)$

Similarly $\rightarrow Q_x = \frac{1 \times 5 + 2 \times 0}{1 + 2} = \frac{5}{3}$

$Q_y = \frac{1 \times 0 + 2 \times 9}{1 + 2} = 6$

$Q : \left(\frac{5}{3}, 6\right)$

Now $m_1 = \frac{3 - 0}{\frac{10}{3} - 0} = \frac{9}{10}$

$m_2 = \frac{6 - 0}{\frac{5}{3} - 0} = \frac{18}{5}$

from (1) & (2)

$9x + 5y = 45$

$9x - 2y = 0$

$\frac{- + -}{7y = 45} \Rightarrow y = \frac{45}{7}$

$\Rightarrow x = \frac{10}{7}$

which satisfy $y - x = 5$ Ans. 4

Question26

Let A(0, 1), B(1, 1) and C(1, 0) be the mid-points of the sides of a triangle with incentre at the point D. If the focus of the parabola

$y^2 = 4ax$ passing through D is $(\alpha + \beta\sqrt{3}, 0)$, where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to
[8-Apr-2023 shift 2]

Options:

A. 6

B. 8

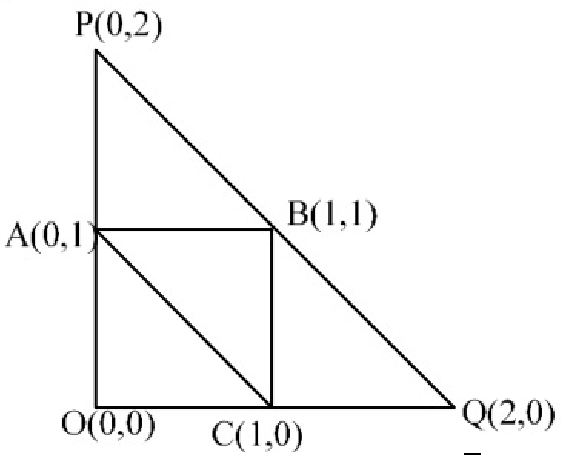
C. $\frac{9}{2}$

D. 12

Answer: B

Solution:

Solution:



$a = OP = 2 \;\; b = OQ = 2 \;\; c = PQ = 2\sqrt{2}$

$D\left(\frac{4}{2+2+2\sqrt{2}}, \frac{4}{2+2+2\sqrt{2}}\right) \equiv D\left(\frac{2}{2+\sqrt{2}}, \frac{2}{2+\sqrt{2}}\right)$

$y^2 = 4ax \Rightarrow \left(\frac{2}{2+\sqrt{2}}\right)^2 = 4a \cdot \left(\frac{2}{2+\sqrt{2}}\right)$

$\therefore 4a = \frac{2}{2+\sqrt{2}} \therefore a = \frac{1}{2} \cdot \frac{2-\sqrt{2}}{4-2} = \frac{1}{4}(2-\sqrt{2})$

$\therefore \alpha = \frac{2}{4} = \frac{1}{2} \;\; \beta = \frac{-1}{4}$

$\therefore \frac{\alpha}{\beta^2} = 8$ Ans.

Question27

The area of the quadrilateral ABCD with vertices $A(2, 1, 1)$, $B(1, 2, 5)$, $C(-2, -3, 5)$ and $D(1, -6, -7)$ is equal to
[8-Apr-2023 shift 2]

Options:

A. 54

B. $9\sqrt{38}$

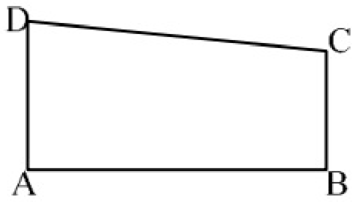
C. 48

D. $8\sqrt{38}$

Answer: D

Solution:

Solution:



Vector Area $= \vec{v}$

$= \frac{1}{2}\vec{AB} \times \vec{AC} + \frac{1}{2}\vec{AC} \times \vec{AD}$

$= \frac{1}{2}(\vec{AB} - \vec{AD}) \times \vec{AC} \quad \left(\begin{array}{c} \vec{AB} = -\hat{i} + \hat{j} + 4\hat{k} \\ \vec{AD} = -\hat{i} - 7\hat{j} - 8\hat{k} \\ \vec{AC} = -4\hat{i} - 4\hat{j} + 4\hat{k} \end{array} \right)$

$= \frac{1}{2}(8\hat{j} + 12\hat{k}) \times (-4)(\hat{i} + \hat{j} - \hat{k})$

$= \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 8 & 12 \\ 1 & 1 & -1 \end{vmatrix}$

$= (-2)(-20\hat{i} + 12\hat{j} - 8\hat{k})$

$= 8(5\hat{i} - 3\hat{j} + 2\hat{k})$

$\therefore$ Area $= |\vec{v}| = 8\sqrt{25 + 9 + 4} = 8\sqrt{38}$ Ans.

Question28

A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius λ. Then the locus of the point, that divides the line segment AB in the ratio 2 : 3, is a circle of radius :
[10-Apr-2023 shift 1]

Options:

A. $\frac{2}{3}\lambda$

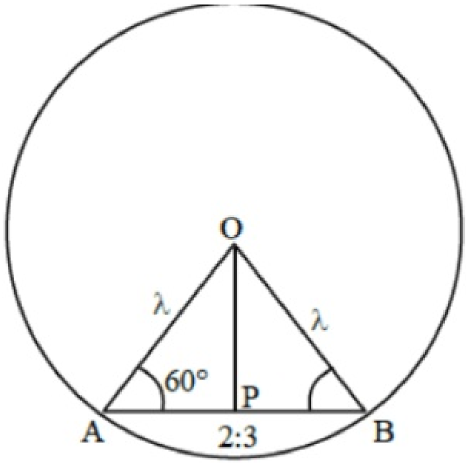
B. $\frac{\sqrt{19}}{7}\lambda$

C. $\frac{3}{5}\lambda$

D. $\frac{\sqrt{19}}{5}\lambda$

Answer: D

Solution:



Since OAB form equilateral Δ

$\therefore \angle OAP = 60^\circ$

$AP = \dfrac{2\lambda}{5}$

$\cos 60^\circ = \dfrac{OA^2 + AP^2 - OP^2}{2\,OA \cdot AP}$

$\Rightarrow \dfrac{1}{2} = \dfrac{\lambda^2 + \dfrac{4\lambda^2}{25} - OP^2}{2\lambda\left(\dfrac{2\lambda}{5}\right)}$

$\Rightarrow \dfrac{2\lambda^2}{5} = \lambda^2 + \dfrac{4\lambda^2}{25} - OP^2$

$\Rightarrow OP\,2 = \dfrac{19\lambda^2}{25}$

$\Rightarrow OP = \dfrac{\sqrt{19}}{5}\lambda$

Therefore, locus of point P is $\dfrac{\sqrt{19}}{5}\lambda$

Question29

Let A be the point (1, 2) and B be any point onthe curve $x^2 + y^2 = 16$. If the centre of the locus of the point P, which divides the line segment AB in the ratio 3 : 2 is the point C(α, β) then the length of the line segment AC is
[10-Apr-2023 shift 2]

Options:

A. $\dfrac{6\sqrt{5}}{5}$

B. $\dfrac{2\sqrt{5}}{5}$

C. $\dfrac{3\sqrt{5}}{5}$

D. $\dfrac{4\sqrt{5}}{5}$

Answer: C

Solution:

$\frac{12\cos\theta + 2}{5} = h \Rightarrow 12\cos\theta = 5h - 2$

sq & add

$144 = (5h - 2)^2 + (5k - 4)^2$

$\left(x - \frac{2}{5}\right)^2 + \left(y - \frac{4}{5}\right)^2 = \frac{144}{25}$

$\text{Centre} \equiv \left(\frac{2}{5}, \frac{4}{5}\right) \equiv (\alpha, \beta)$

$AC = \sqrt{\left(1 - \frac{2}{5}\right)^2 + \left(2 - \frac{4}{5}\right)^2}$

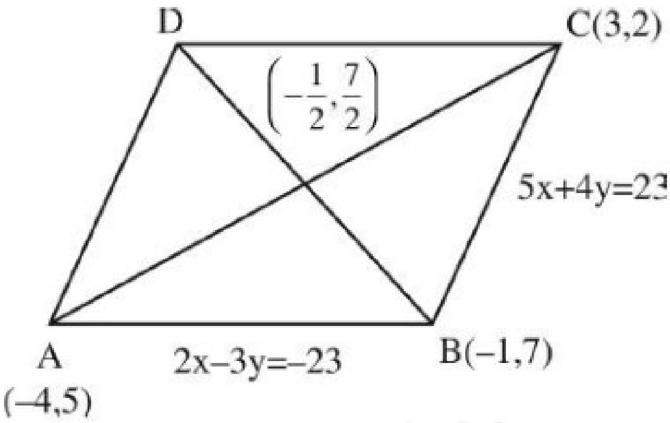
$= \sqrt{\frac{9}{25} + \frac{36}{25}} = \frac{\sqrt{45}}{5} = \frac{3\sqrt{5}}{5}$

Question30

Let the equations of two adjacent sides of a parallelogram ABCD be $2x - 3y = -23$ and $5x + 4y = 23$. If the equation of its one diagonal AC is $3x + 7y = 23$ and the distance of A from the other diagonal is d , then $50d^2$ is equal to _______.
[10-Apr-2023 shift 2]

Answer: 529

Solution:



A & C point will be $(-4, 5)$&$(3, 2)$

mid point of AC will be $\left(-\frac{1}{2}, \frac{7}{2}\right)$

equation of diagonal BD is

$y - \frac{7}{2} = \frac{\frac{7}{2}}{-\frac{1}{2}}\left(x + \frac{1}{2}\right)$

$\Rightarrow 7x + y = 0$

Distance of A from diagonal BD

$= d = \frac{23}{\sqrt{50}}$

$\Rightarrow \quad 50d^{2} = (23)^{2}$
$50d^{2} = 529$

Question31

Let R be a rectangle given by the line x = 0, x = 2, y = 0 and y = 5. Let A(α, 0) and B(0, β), α ∈ [0, 2] and β ∈ [0, 5], be such that the line segment AB divides the area of the rectangle R in the ratio 4 : 1. Then, the midpoint of AB lies on a :
[11-Apr-2023 shift 1]

Options:

A. straight line

B. parabola

C. circle

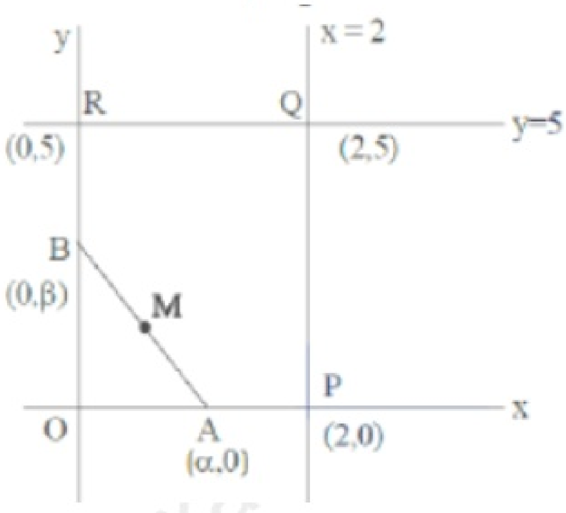
D. hyperbola

Answer: D

Solution:

$$\frac{ar(OPQR)}{or(OAB)} = \frac{4}{1}$$

Let M be the mid-point of AB.



$$M(h, k) \equiv \left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$$

$$\Rightarrow \frac{10 - \frac{1}{2}\alpha\beta}{\frac{1}{2}\alpha\beta} = 4$$

$$\Rightarrow \frac{5}{2}\alpha\beta = 10 \Rightarrow \alpha\beta = 4$$

$\Rightarrow (2h)(2K) = 4$

∴ Locus of M is $xy = 1$

Which is a hyperbola.

Question32

If the line l_1 : 3y − 2x = 3 is the angular bisector of the line l_2 : x − y + 1 = 0 and l_3 : αx + βy + 17, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to _______.
[11-Apr-2023 shift 2]

Answer: 348

Solution:

Point of intersection of $\ell_1 : 3y - 2x = 3$
$\ell_2 : x - y + 1 = 0$ is $P \equiv (0, 1)$
Which lies on $\ell_3 : \alpha x - \beta y + 17 = 0$,
$\Rightarrow \beta = -17$
Consider a random point $Q \equiv (-1, 0)$
on $\ell_2 : x - y + 1 = 0$, image of Q about
$\ell_2 : x - y + 1 = 0$, is $Q' \equiv \left(\frac{-17}{13}, \frac{6}{13}\right)$ which is calculated by formulae
$\frac{x - (-1)}{2} = \frac{y - 0}{-3} = 2\left(\frac{-2 + 3}{13}\right)$
Now, Q' lies in $\ell_3 : \alpha x + \beta y + 17 = 0$
$\Rightarrow \alpha = 7$
Now, $\alpha^2 + \beta^2 - \alpha - \beta = 348$

Question33

If the point $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ lies on the curve traced by the mid-points of the line segments of the lines $x\cos\theta + y\sin\theta = 7$, $\theta \in \left(0, \frac{\pi}{2}\right)$ between the co-ordinates axes, then α is equal to
[12-Apr-2023 shift 1]

Options:

A. $7\sqrt{3}$

B. -7

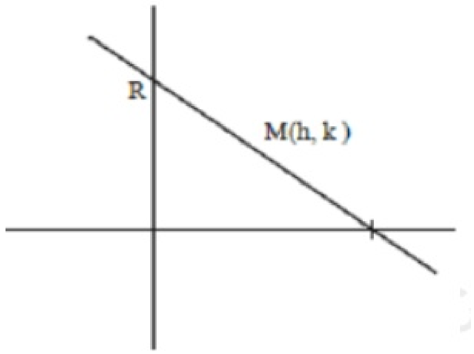
C. 7

D. $-7\sqrt{3}$

Answer: C

Solution:

pt $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$



$x\cos\theta + y\sin\theta = 7$

x - intercept $= \frac{7}{\cos\theta}$

y − intercept $= \frac{7}{\sin\theta}$

A : $\left(\frac{7}{\cos\theta}, 0\right)$ B : $\left(0, \frac{7}{\sin\theta}\right)$

Locus of mid pt M : (h, k)

$h = \frac{7}{2\cos\theta}, k = \frac{7}{2\sin\theta}$

$\frac{7}{2\sin\theta} = \frac{7\sqrt{3}}{3} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$

$\alpha = \frac{7}{2\cos\theta} = 7$

Question34

Let (α, β) be the centroid of the triangle formed by the lines $15x - y = 82$, $6x - 5y = -4$ and $9x + 4y = 17$. Then $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots of the equation [13-Apr-2023 shift 2]

Options:

A. $x^2 - 13x + 42 = 0$

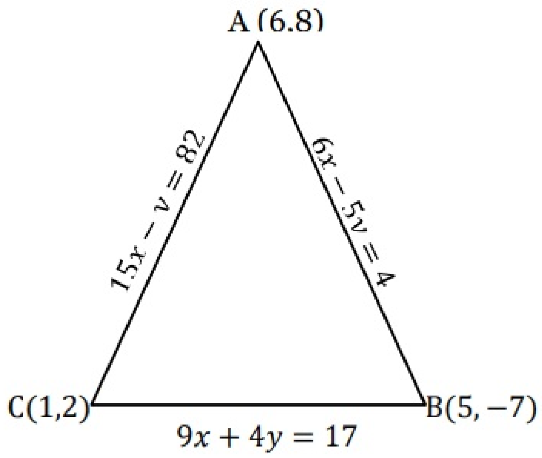
B. $x^2 - 10x + 25 = 0$

C. $x^2 - 7x + 12 = 0$

D. $x^2 - 14x + 48 = 0$

Answer: A

Solution:



Centroid $(\alpha, \beta) = \left(\frac{6+1+5}{3}, \frac{8-7+2}{3}\right) = (4, 1)$

$\alpha + 2\beta = 4 + 2 = 6$
$2\alpha - \beta = 8 - 1 = 7$
Quadratic equation
$x^2 - (6+7)x + (6 \times 7) = 0$
$\Rightarrow x^2 - 13x + 42 = 0$

Question35

If (α, β) is the orthocenter of the triangle ABC with vertices A(3, −7), B(−1, 2) and C(4, 5), then $9\alpha - 6\beta + 60$ is equal to [15-Apr-2023 shift 1]

Options:

A. 30

B. 35

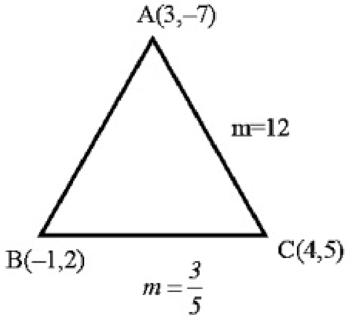
C. 40

D. 25

Answer: D

Solution:

Solution:



Altitude of BC : $y + 7 = \frac{-5}{3}(x - 3)$

$3y + 21 = -5x + 15$
$5x + 3y + 6 = 0$

Altitude of AC: $y - 2 = \frac{-1}{12}(x + 1)$

$12y - 24 = -x - 1$
$x + 12y = 23$

$\alpha = \frac{-47}{19}, \ \beta = \frac{121}{57}$
$9\alpha - 6\beta + 60 = 25$

Question36

From the top A of a vertical wall AB of height 30m, the angles of depression of the top P and bottom Q of a vertical tower PQ are 15° and 60° respectively, B and Q are on the same horizontal level. If C is a point on AB such that CB = PQ, then the area (in m^2) of the quadrilateral BCPQ is equal to :
[6-Apr-2023 shift 1]

Options:

A. $200(3 - \sqrt{3})$

B. $300(\sqrt{3} + 1)$

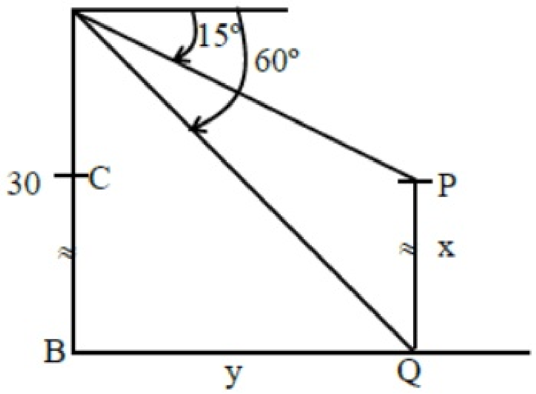
C. $300(\sqrt{3} - 1)$

D. $600(\sqrt{3} - 1)$

Answer: D

Solution:

Solution:



$\frac{AB}{BQ} = \tan 60^{\circ}$

$BQ = \frac{30}{\sqrt{3}} = 10\sqrt{3} = y$

& $\triangle$ ACP

$\frac{AC}{CP} = \tan 15^{\circ} \Rightarrow \frac{(30 - x)}{y} = (2 - \sqrt{3})$

$30 - x = 10\sqrt{3}(2 - \sqrt{3})$

$30 - x = 20\sqrt{3} - 30$

$x = 60 - 20\sqrt{3}$

$\text{Area} = x \cdot y = 20(3 - \sqrt{3}) \cdot 10\sqrt{3}$

$= 600(\sqrt{3} - 1)$

Question37

The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is 45° and from the feet of another person standing due west of the tower is 30°. If the height of the tower is 5 meters, then the distance (in meters) between the two persons is equal to
[11-Apr-2023 shift 2]

Options:

A. 10

B. $5\sqrt{5}$

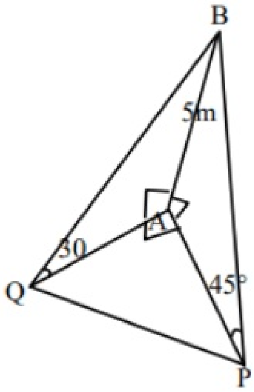
C. $\frac{5}{2}\sqrt{5}$

D. 5

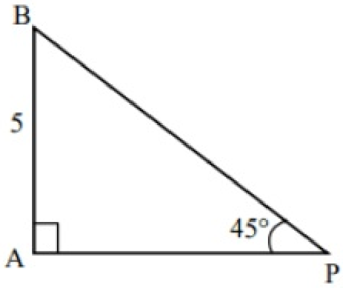
Answer: A

Solution:

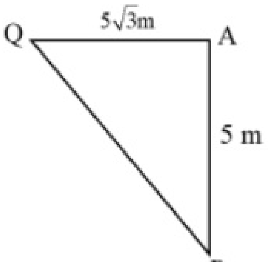
Solution:



Tower AB = 5m
$\angle APB = 45^{\circ}$
$\angle PAB = 90^{\circ}$

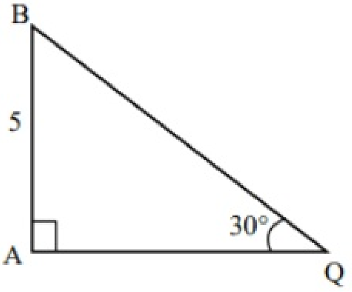


$\tan 45^{\circ} = \frac{AB}{AP}$
$1 = \frac{AB}{AP}$
AP = 5m



$\tan 30^{\circ} = \frac{AB}{AQ}$

$\frac{1}{1\sqrt{3}} = \frac{5}{AQ}$
$AQ = 5\sqrt{3}$



$AP^2 + AQ^2 = PQ^2$
$PQ^2 = 5^2 + (5\sqrt{3})^2$
$PQ^2 = 25 + 75 = 100$
$PQ = 10\,cm$
Option (A) 10 cm correct.

Question38

Let $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$, $a > 0$, be a fixed point in the xy-plane. The image of A in y-axis be B and the image of B in x-axis be C. If $D(3\cos\theta, a\sin\theta)$ is a point in the fourth quadrant such that the maximum area of $\triangle ACD$ is 12 square units, then a is equal to____
[24-Jun-2022-Shift-1]

Let $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$, $a > 0$, be a fixed point in the xy-plane. The image of A in y-axis be B and the image of B in x-axis be C. If $D(3\cos\theta, a\sin\theta)$ is a point in the fourth quadrant such that the maximum area of $\triangle ACD$ is 12 square units, then a is equal to____
[24-Jun-2022-Shift-1]

Answer: 8

Solution:

Clearly B is $\left(-\frac{3}{\sqrt{a}}, +\sqrt{a}\right)$ and C is $\left(-\frac{3}{\sqrt{a}}, -\sqrt{a}\right)$

$$\text{Area of } \triangle ACD = \frac{1}{2}\begin{vmatrix} \frac{3}{\sqrt{a}} & \sqrt{a} & 1 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & 0 & 1 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = |3\sqrt{a}\sin\theta + 3\sqrt{a}\cos\theta| = 3\sqrt{a}\,|\sin\theta + \cos\theta|$$

$$\Rightarrow \Delta_{max} = 3\sqrt{a}\cdot\sqrt{2} = 12 \Rightarrow a = (2\sqrt{2})^2 = 8$$

Question39

Let the area of the triangle with vertices A(1, α), B(α, 0) and C(0, α) be 4 sq. units. If the points (α, − α), (−α, α) and (α^2, β) are collinear, then β is equal to
[24-Jun-2022-Shift-2]

Options:

A. 64

B. −8

C. −64

D. 512

Answer: C

Solution:

$\because A(1, \alpha), B(\alpha, 0)$ and $C(0, \alpha)$ are the vertices of ΔABC and area of $\Delta ABC = 4$

$$\therefore \left| \frac{1}{2} \begin{vmatrix} 1 & \alpha & 1 \\ \alpha & 0 & 1 \\ 0 & \alpha & 1 \end{vmatrix} \right| = 4$$

$$\Rightarrow |\ 1(1-\alpha) - \alpha(\alpha) + \alpha^2\ | = 8$$

$$\Rightarrow \alpha = \pm 8$$

Now, $(\alpha, -\alpha)$, $(-\alpha, \alpha)$ and (α^2, β) are collinear

$$\therefore \begin{vmatrix} 8 & -8 & 1 \\ -8 & 8 & 1 \\ 64 & \beta & 1 \end{vmatrix} = 0 = \begin{vmatrix} -8 & 8 & 1 \\ 8 & -8 & 1 \\ 64 & \beta & 1 \end{vmatrix}$$

$$\Rightarrow 8(8-\beta) + 8(-8-64) + 1(-8\beta - 8 \times 64) = 0$$

$$\Rightarrow 8 - \beta - 72 - \beta - 64 = 0$$

$$\Rightarrow \beta = -64$$

Question40

Let R be the point (3, 7) and let P and Q be two points on the line $x + y = 5$ such that PQR is an equilateral triangle. Then the area of $\triangle$PQR is :
[26-Jun-2022-Shift-1]

Options:

A. $\frac{25}{4\sqrt{3}}$

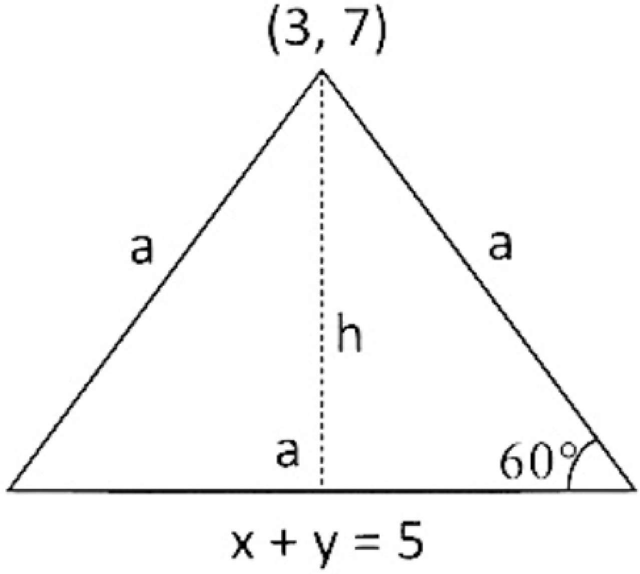
B. $\frac{25\sqrt{3}}{2}$

C. $\frac{25}{\sqrt{3}}$

D. $\frac{25}{2\sqrt{3}}$

Answer: D

Solution:



Let, side of triangle $= a$.

$$h = \frac{|3+7-5|}{\sqrt{1^2+1^2}}$$

$$= \frac{5}{\sqrt{2}}$$

From figure, $h = a \sin 60^\circ$

$$\Rightarrow a = \frac{2h}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{5}{\sqrt{2}}$$

$$= \frac{10}{\sqrt{6}}$$

$$\therefore \text{ Area } = \frac{3}{4}\left(\frac{10}{\sqrt{6}}\right)^2$$

$$= \frac{25}{2\sqrt{3}}$$

Question41

In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is 2x + y = 4. Let the point B lie on the line x+ 3y = 7. If (α, β) is the centroid of △ABC, then 15(α + β) is equal to:
[27-Jun-2022-Shift-1]

Options:

A. 39

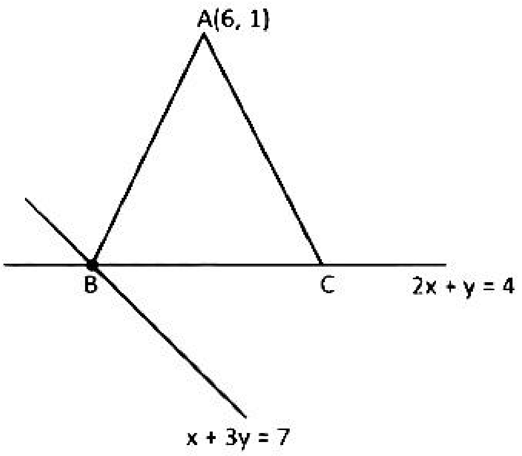
B. 41

C. 51

D. 63

Answer: C

Solution:



$$\left.\begin{array}{r} 2x + y = 4 \\ 2x + 6y = 14 \end{array}\right\} y = 2, x = 3$$

B(1, 2)

Let C(k, 4 − 2k)

Now $AB^2 = AC^2$

$5^2 + (-1)^2 = (6 - k)^2 + (-3 + 2k)^2$

$\Rightarrow 5k^2 - 24k + 19 = 0$

$(5k - 19)(k - 1) = 0 \Rightarrow k = \frac{19}{5}$

$C\left(\frac{19}{5}, -\frac{18}{5}\right)$

Centroid (α, β)

$$\alpha = \frac{6 + 1 + \frac{19}{5}}{3} = \frac{18}{5}$$

$$\beta = \frac{1 + 2 - \frac{18}{5}}{3} = -\frac{1}{5}$$

Now 15(α + β)

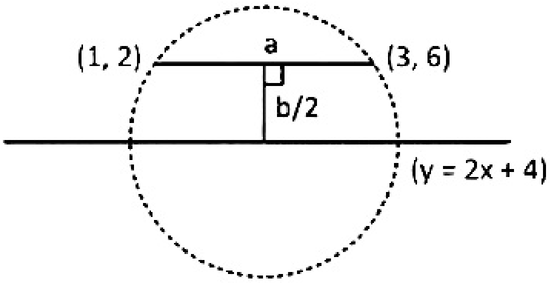
$15\left(\frac{17}{5}\right) = 51$

Question42

A rectangle R with end points of one of its sides as (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of the circle is $2x - y + 4 = 0$, then the area of R is
[27-Jun-2022-Shift-1]

Answer: 16

Solution:



As slope of line joining (1, 2) and (3, 6) is 2 given diameter is parallel to side

$\therefore a = \sqrt{(3-1)^2 + (6-2)^2} = \sqrt{20}$

and $b / 2 = \frac{4}{\sqrt{5}} \Rightarrow b = \frac{8}{\sqrt{5}}$

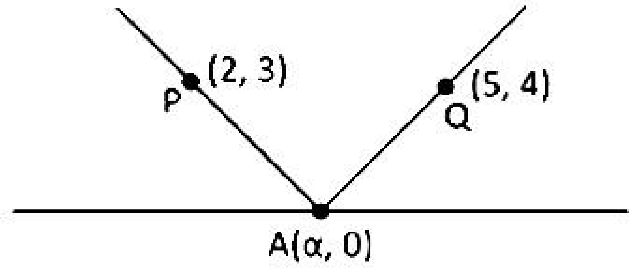
$\text{Area} = ab = 2\sqrt{5} \cdot \frac{8}{\sqrt{5}} = 16$

Question43

A ray of light passing through the point P(2, 3) reflects on the x-axis at point A and the reflected ray passes through the point Q(5, 4). Let R be the point that divides the line segment AQ internally into the ratio 2 : 1. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β). Then, the value of 7α + 3β is equal to
[28-Jun-2022-Shift-1]

Answer: 31

Solution:



$\frac{4}{5-\alpha} = \frac{3}{\alpha - 2} \Rightarrow 4\alpha - 8 = 15 - 3\alpha$

$\alpha = \frac{23}{7}$

$A = \left(\frac{23}{7}, 0\right) Q = (5, 4)$

$R = \left(\frac{10 + \frac{23}{7}}{3}, \frac{8}{3}\right)$

$= \left(\frac{31}{7}, \frac{8}{3}\right)$

Bisector of angle PAQ is $X = \frac{23}{7}$

$\Rightarrow M = \left(\frac{23}{7}, \frac{8}{3}\right)$

So, $7\alpha + 3\beta = 31$

Question44

Let a triangle be bounded by the lines
$L_1 : 2x + 5y = 10$; $L_2 : -4x + 3y = 12$ and the line L_3, which passes through the point P(2, 3), intersects L_2 at A and L_1 at B. If the point P divides the line-segment AB, internally in the ratio 1 : 3, then the area of the triangle is equal to :
[28-Jun-2022-Shift-2]

Options:

A. $\frac{110}{13}$

B. $\frac{132}{13}$

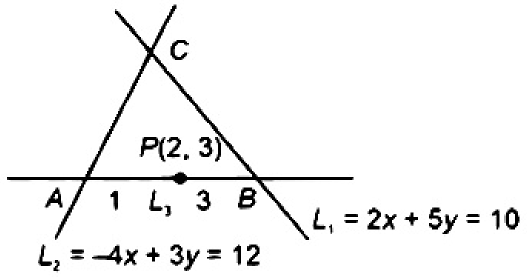
C. $\frac{142}{13}$

D. $\frac{151}{13}$

Answer: B

Solution:

Solution:
$L_1 : 2x + 5y = 10$
$L_2 : -4x + 3y = 12$



Solving L_1 and L_2 we get

$C \equiv \left(\frac{-15}{13}, \frac{32}{13} \right)$

Now, Let $A\left(x_1, \frac{1}{3}(12 + 4x_1) \right)$ and

$B\left(x_2, \frac{1}{5}(10 - 2x_2) \right)$

$\therefore \frac{3x_1 + x_2}{4} = 2$

and $\frac{(12 + 4x_1) + \frac{10 - 2x_2}{5}}{4} = 3$

So, $3x_1 + x_2 = 8$ and $10x_1 - x_2 = -5$

So, $(x_1, x_2) = \left(\frac{3}{13}, \frac{95}{13} \right)$

$A = \left(\frac{3}{13}, \frac{56}{13} \right)$ and $B = \left(\frac{95}{13}, \frac{-12}{13} \right)$

$= \left| \frac{1}{2}\left(\frac{3}{13}\left(\frac{-44}{13} \right) \frac{-56}{13}\left(\frac{110}{13} \right) + 1\left(\frac{2860}{169} \right) \right) \cdot \right|$

$= \frac{132}{13}$ sq. units

Question45

The distance between the two points A and A' which lie on $y = 2$ such that both the line segments AB and $A'B$ (where B is the point (2, 3)) subtend angle $\frac{\pi}{4}$ at the origin, is equal to:
[29-Jun-2022-Shift-1]

Options:

A. 10

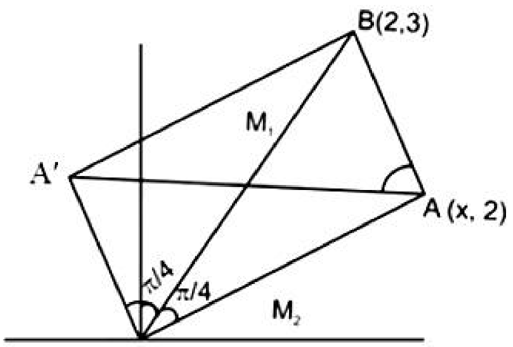
B. $\frac{48}{5}$

C. $\frac{52}{5}$

D. 3

Answer: C

Solution:



$M_1 = 3/2 \quad M_2 = 2/x$

$\tan \pi/4 = \left| \frac{3/2 - 2/x}{1 + 6/2x} \right| = 1$

$\Rightarrow x_1 = 10, \quad x_2 = -2/5$

$\Rightarrow AA^1 = 52/5$

Question46

The distance of the origin from the centroid of the triangle whose two sides have the equations $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $\left(\frac{7}{3}, \frac{7}{3}\right)$ is :
[29-Jun-2022-Shift-2]

Options:

A. $\sqrt{2}$

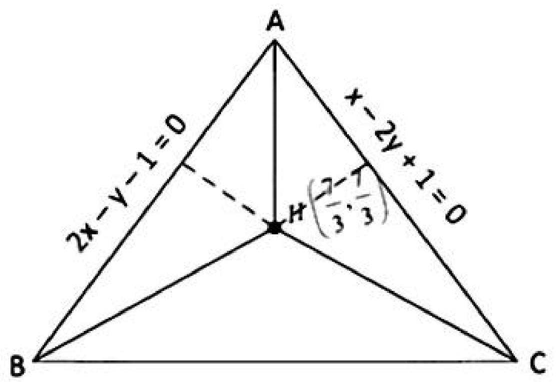
B. 2

C. $2\sqrt{2}$

D. 4

Answer: C

Solution:



For point A,
$2x - y - 1 = 0$
$x - 2y + 1 = 0$
Solving (1) and (2), we get
$x = 1, y = 1$
∴ Point $A = (1, 1)$
Altitude from B to line AC is perpendicular to line AC.
∴ Equator of altitude BH is
$2x + y + \lambda = 0$
It passes through point $H\left(\frac{7}{3}, \frac{7}{3}\right)$ so it satisfy the equation (3).
$\frac{14}{3} + \frac{7}{3} + \lambda = 0$
$\Rightarrow \alpha = -7$
∴ Altitude $BH = 2x + y - 7 = 0$
Solving equation (1) and (4), we get
$x = 2, y = 3$
∴ Point $B = (2, 3)$
Altitude from C to line AB is perpendicular to line AB.
∴ Equation of altitude CH is
$x + 2y + \lambda = 0$
It passes through point $H\left(\frac{7}{3}, \frac{7}{3}\right)$ so it satisfy equation (5).
$\frac{7}{3} + \frac{14}{3} + \lambda = 0$
$\Rightarrow \lambda = -7$
∴ Altitude $CH = x + 2y - 7 = 0$
Solving equation (2) and (6), we get
$x = 3, y = 2$
∴ Point $C = (3, 2)$
Centroid $G(x, y)$ of triangle $A(1, 1)$, $B(2, 3)$ and $C(3, 2)$ is
$x = \frac{1+2+3}{3} = 2, y = \frac{1+2+3}{3} = 2$
Now d Distance of point $G(2, 2)$ from center $O(0, 0)$ is
$OG = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

Question47

Let AB and PQ be two vertical poles, 160m apart from each other. Let C be the middle point of B and Q, which are feet of these two poles. Let $\frac{\pi}{8}$ and θ be the angles of elevation from C to P and A, respectively. If the height of pole PQ is twice the height of pole AB, then $\tan^2\theta$ is equal to [28-Jun-2022-Shift-1]

Options:

A. $\frac{3-2\sqrt{2}}{2}$

B. $\frac{3+\sqrt{2}}{2}$

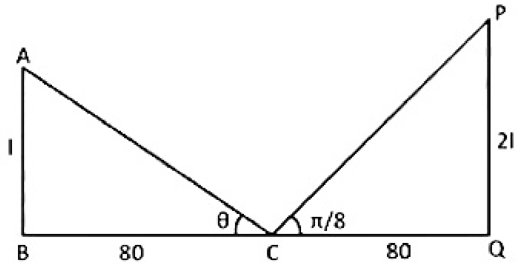
C. $\frac{3-2\sqrt{2}}{4}$

D. $\frac{3-\sqrt{2}}{4}$

Answer: C

Solution:

Solution:



$\frac{l}{80} = \tan\theta \ldots.$ (i)

$\frac{2l}{80} = \tan\frac{\pi}{8} \ldots.$ (ii)

From (i) and (ii)

$\frac{1}{2} = \frac{\tan\theta}{\tan\frac{\pi}{8}} \Rightarrow \tan^2\theta = \frac{1}{4}\tan^2\frac{\pi}{8}$

$\Rightarrow \tan^2\theta = \frac{\sqrt{2}-1}{4(\sqrt{2}+1)} = \frac{3-2\sqrt{2}}{4}$

Question48

From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60°. The pole subtends an angle 30° at the top of the tower. Then the height of the tower is
[29-Jun-2022-Shift-2]

Options:

A. $15\sqrt{3}$

B. $20\sqrt{3}$

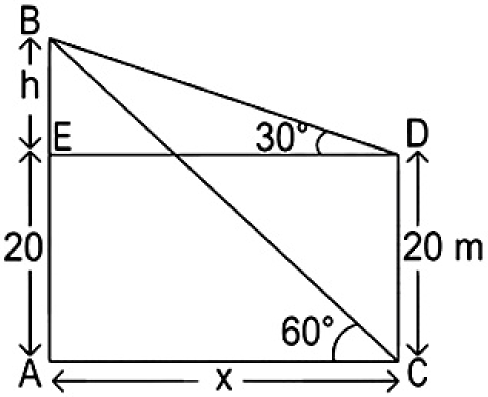
C. $20+10\sqrt{3}$

D. 30

Answer: D

Solution:

Solution:
Here AB is a tower and CD is a pole.
In triangle ABC, $\tan 60^\circ = \frac{AB}{AC} = \frac{20+h}{x} \ldots\ldots(1)$
In triangle BED, $\tan 30^\circ = \frac{h}{x} \ldots\ldots(2)$
Divide equation (1) by equation (2), we get
$\frac{\tan 60^\circ}{\tan 30^\circ} = \frac{20+h}{x} \times \frac{x}{h}$
$\Rightarrow \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = \frac{20+h}{h}$
$\Rightarrow 3 = \frac{20+h}{h}$
$\Rightarrow 3h = 20 + h$
$\Rightarrow h = 10m$
$\therefore$ Height of tower $= 20 + 10 = 30m$



Question49

A line, with the slope greater than one, passes through the point A(4, 3) and intersects the line $x - y - 2 = 0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line:
[25-Jul-2022-Shift-1]

Options:

A. $2x + y = 9$

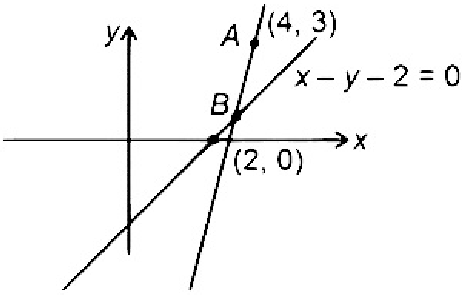
B. $3x - 2y = 7$

C. $x + 2y = 6$

D. $2x - 3y = 3$

Answer: C

Solution:



Let inclination of required line is θ,
So the coordinates of point B can be assumed as
$\left(4 - \frac{\sqrt{29}}{3}\cos\theta, 3 - \frac{\sqrt{29}}{3}\sin\theta\right)$
Which satisfices $x - y - 2 = 0$
$4 - \frac{\sqrt{29}}{3}\cos\theta - 3 + \frac{\sqrt{29}}{3}\sin\theta - 2 = 0$
$\sin\theta - \cos\theta = \frac{3}{\sqrt{29}}$
By squaring
$\sin 2\theta = \frac{20}{29} = \frac{2\tan\theta}{1 + \tan^2\theta}$
$\tan\theta = \frac{5}{2}$ only (because slope is greater than 1)
$\sin\theta = \frac{5}{\sqrt{29}}, \cos\theta = \frac{2}{\sqrt{29}}$
Point B : $\left(\frac{10}{3}, \frac{4}{3}\right)$
Which also satisfies $x + 2y = 6$

Question50

Let the point $P(\alpha, \beta)$ be at a unit distance from each of the two lines $L_1 : 3x - 4y + 12 = 0$, and $L_2 : 8x + 6y + 11 = 0$. If P lies below L_1 and above L_2, then $100(\alpha + \beta)$ is equal to
[25-Jul-2022-Shift-2]

Options:

A. -14

B. 42

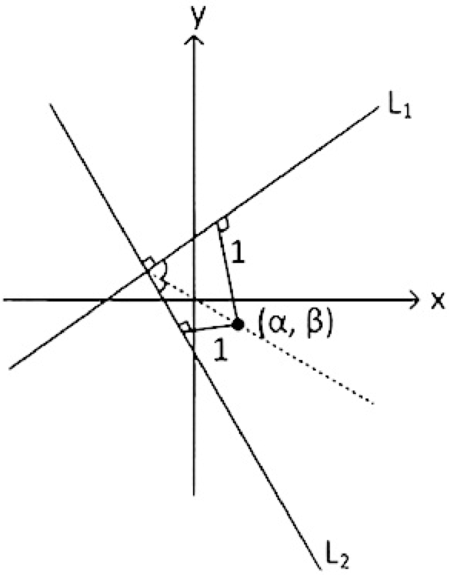
C. -22

D. 14

Answer: D

Solution:

Solution:



$L_1 : 3x - 4y + 12 = 0$
$L_2 : 8x + 6y + 11 = 0$
Equation of angle bisector of L_1 and L_2 of angle containing origin
$2(3x - 4y + 12) = 8x + 6y + 11$
$2x + 14y - 13 = 0 \ldots\ldots.$ (i)
$\frac{3\alpha - 4\beta + 12}{5} = 1$
$\Rightarrow 3\alpha - 4\beta + 7 = 0$ ..... (ii)
Solution of $2x + 14y - 13 = 0$ and $3x - 4y + 7 = 0$ gives the required point $P(\alpha, \beta)$, $\alpha = \frac{-23}{25}$, $\beta = \frac{53}{50}$ $100(\alpha + \beta) = 14$

Question51

A point P moves so that the sum of squares of its distances from the points (1, 2) and (−2, 1) is 14 . Let f (x, y) = 0 be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the points C, D. Then the area of the quadrilateral ACBD is equal to :
[26-Jul-2022-Shift-1]

Options:

A. $\frac{9}{2}$

B. $\frac{3\sqrt{17}}{2}$

C. $\frac{3\sqrt{17}}{4}$

D. 9

Answer: B

Solution:

Solution:
Let point P : (h, k)
$(h - 1)^2 + (k - 2)^2 + (h + 2)^2 + (k - 1)^2 = 14$
$2h^2 + 2k^2 + 2h - 6k - 4 = 0$
Locus of P : $x^2 + y^2 + x - 3y - 2 = 0$
Intersection with x -axis,
$x^2 + x - 2 = 0$

$\Rightarrow x = -2, 1$

Intersection with y-axis,
$y^2 - 3y - 2 = 0$
$\Rightarrow y = \frac{3 \pm \sqrt{17}}{2}$
Area of the quadrilateral ACBD is
$= \frac{1}{2}(|x_1| + |x_2|)(|y_1| + |y_2|)$
$= \frac{1}{2} \times 3 \times \sqrt{17} = \frac{3\sqrt{17}}{2}$

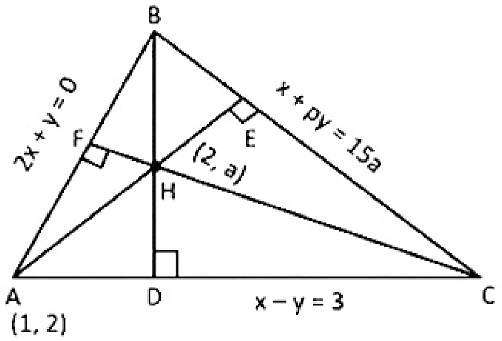
Question52

The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 15a$ and $x - y = 3$ respectively. If its orthocentre is $(2, a)$, $-\frac{1}{2} < a < 2$, then p is equal to ________.
[26-Jul-2022-Shift-1]

Answer: 3

Solution:

Solution:



Slope of AH $= \frac{a+2}{1}$
Slope of BC $= -\frac{1}{p}$
$\therefore p = a + 2 \ldots\ldots$ (i)
Coordinate of C $= \left(\frac{18p - 30}{p + 1}, \frac{15p - 33}{p + 1} \right)$
Slope of H C $= \frac{\frac{15p - 33}{p + 3} - a}{\frac{18p - 33}{p + 1} - 2} = \frac{15p - 33 - (p - 2)(p + 1)}{18p - 30 - 2p - 2}$
$= \frac{16p - p^2 - 31}{16p - 32}$
$\because \frac{16p - p^2 - 31}{16p - 32} \times -2 = -1$
$\therefore p^2 - 8p + 15 = 0$
$\therefore p = 3$ or 5
But if $p = 5$ then $a = 3$ not acceptable
$\therefore p = 3$

Question53

Let A(1, 1), B(−4, 3), C(−2, −5) be vertices of a triangle ABC, P be a point on side BC, and Δ_1 and Δ_2 be the areas of triangles APB and ABC, respectively. If $\Delta_1 : \Delta_2 = 4 : 7$, then the area enclosed by the lines AP, AC and the x-axis is
[27-Jul-2022-Shift-1]

Options:

A. $\frac{1}{4}$

B. $\frac{3}{4}$

C. $\frac{1}{2}$

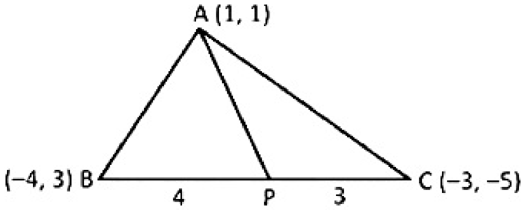
D. 1

Answer: C

Solution:

Solution:

$\frac{\Delta_1}{\Delta_2} = \frac{\frac{1}{2} \times BP \times AH}{\frac{1}{2} \times BC \times AH} = \frac{4}{7}$



$P\left(\frac{-20}{7}, \frac{-11}{7}\right)$

Line AC : $y - 1 = 2(x - 1)$

Intersection with x -axis $= \left(\frac{1}{2}, 0\right)$

Line AP : $y - 1 = \frac{2}{3}(x - 1)$

Intersection with x-axis $\left(\frac{-1}{2}, 0\right)$

Vertices are $(1, 1)$, $\left(\frac{1}{2}, 0\right)$ and $\left(\frac{-1}{2}, 0\right)$

Area $= \frac{1}{2}$ sq. unit

Question54

The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 39$ and $x - y = 3$ respectively and P(2, 3) is its circumcentre. Then which of the following is NOT true?
[27-Jul-2022-Shift-2]

Options:

A. $(AC)^2 = 9p$

B. $(AC)^2 + p^2 = 136$

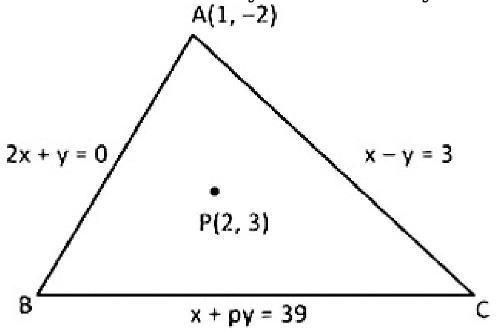
C. $32 < \text{area}(\Delta ABC) < 36$

D. $34 < \text{area}(\triangle ABC) < 38$

Answer: D

Solution:

Solution:
Intersection of $2x + y = 0$ and $x - y = 3$: $A(1, -2)$



Equation of perpendicular bisector of AB is
$x - 2y = -4$
Equation of perpendicular bisector of AC is
$x + y = 5$
Point B is the image of A in line $x - 2y + 4 = 0$ which is obtained as $B\left(\frac{-13}{5}, \frac{26}{5}\right)$
Similarly vertex C : $(7, 4)$
Equation of line BC : $x + 8y = 39$
So, $p = 8$
$AC = \sqrt{(7-1)^2 + (4+2)^2} = 6\sqrt{2}$
Area of triangle ABC = 32.4

Question55

For $t \in (0, 2\pi)$, if ABC is an equilateral triangle with vertices $A(\sin t, -\cos t)$, $B(\cos t, \sin t)$ and $C(a, b)$ such that its orthocentre lies on a circle with centre $\left(1, \frac{1}{3}\right)$, then $(a^2 - b^2)$ is equal to:

[28-Jul-2022-Shift-1]

Options:

A. $\frac{8}{3}$

B. 8

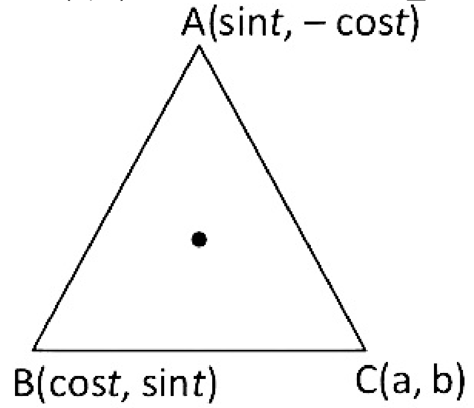
C. $\frac{77}{9}$

D. $\frac{80}{9}$

Answer: B

Solution:

Solution:
Let P(h, k) be the orthocentre of $\triangle ABC$



Then
$h = \frac{\sin t + \cos t + a}{3}, k = \frac{-\cos t + \sin t + b}{3}$
(orthocentre coincide with centroid)
$\therefore (3h - a)^2 + (3k - b)^2 = 2$
$\therefore \left(h - \frac{a}{3}\right)^2 + \left(k - \frac{b}{3}\right)^2 = \frac{2}{9}$
$\because$ orthocentre lies on circle with centre $\left(1, \frac{1}{3}\right)$
$\therefore a = 3, b = 1$
$\therefore a^2 - b^2 = 8$

Question56

Let the circumcentre of a triangle with vertices A(a, 3), B(b, 5) and C(a, b), ab > 0 be P(1, 1). If the line AP intersects the line BC at the point $Q(k_1, k_2)$, then $k_1 + k_2$ is equal to :
[29-Jul-2022-Shift-1]

Options:

A. 2

B. $\frac{4}{7}$

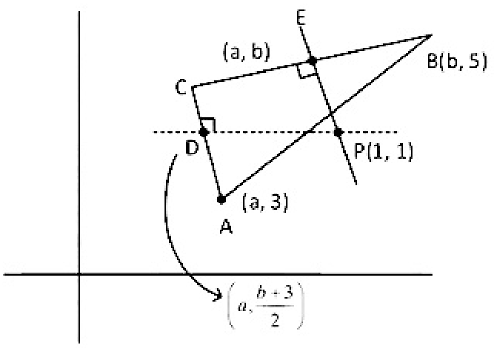
C. $\frac{2}{7}$

D. 4

Answer: B

Solution:

Solution:



Let D be mid-point of AC, then

$\frac{b+3}{2} = 1 \Rightarrow b = -1$

Let E be mid-point of BC,

$\frac{5-b}{b-a} \cdot \frac{\frac{(3+b)}{2}}{\frac{a+b}{2} - 1} = -1$

On putting $b = -1$, we get $a = 5$ or -3

But $a = 5$ is rejected as $ab > 0$

$A(-3, 3), B(-1, 5), C(-3, -1), P(1, 1)$

Line BC $\Rightarrow y = 3x + 8$

Line AP $\Rightarrow y = \frac{3-x}{2}$

Point of intersection $\left(\frac{-13}{7}, \frac{17}{7}\right)$

Question57

Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2 + 11a + 3(m_1^2 + m_2^2) = 220$. If one vertex of the square is $(10(\cos\alpha - \sin\alpha), 10(\sin\alpha + \cos\alpha))$, where $\alpha \in \left(0, \frac{\pi}{2}\right)$ and the equation of one diagonal is $(\cos\alpha - \sin\alpha)x + (\sin\alpha + \cos\alpha)y - 10$, then $72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$ is equal to: [29-Jul-2022-Shift-2]

Options:

A. 119

B. 128

C. 145

D. 155

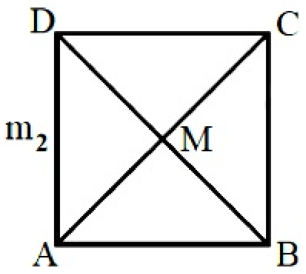
Answer: B

Solution:

Solution:

$m_1 m_2 = -1$

$a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1^2}\right) = 220$



Eq. of AC

$AC = (\cos\alpha - \sin\alpha) + (\sin\alpha + \cos\alpha)y = 10$

$BD = (\sin\alpha - \cos\alpha)x + (\sin\alpha - \cos\alpha)y = 0$

$(10(\cos\alpha - \sin\alpha), 10(\sin\alpha - \cos\alpha))$

Slope of AC $= \left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right) = \tan\theta = M$

Eq. of line making an angle π_4 with AC

$$m_1, m_2 = \frac{m \pm \tan\frac{\pi}{4}}{1 \pm m\tan\frac{\pi}{4}}$$

$$= \frac{m+1}{1-m} \text{ or } \frac{m-1}{1+m}$$

$$\frac{\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} + 1}{1 - \left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right)}, \frac{\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} - 1}{1 + \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}}$$

$m_1, m_2 = \tan\alpha, \cot\alpha$

mid point of $AC \& BD$

$= M(5(\cos\alpha - \sin\alpha), 5(\cos\alpha + \sin\alpha))$

B $(10(\cos\alpha - \sin\alpha), 10(\cos\alpha + \sin\alpha))$

$a = AB = \sqrt{2}BM = \sqrt{2}(5\sqrt{2}) = 10$

$a = 10$

$$\because a^2 + 11a + 3\left(m_1{}^2 + \frac{1}{m_1 2}\right) = 220$$

$100 + 110 + 3(\tan^2\alpha + \cot^2\alpha) = 220$

Hence $\tan^2\alpha = 3, \tan^2\alpha = \frac{1}{3} \Rightarrow \alpha = \frac{\pi}{3}$ or $\frac{\pi}{6}$

Now $72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$

$$= 72\left(\frac{9}{16} + \frac{1}{16}\right) + 100 - 30 + 13$$

$$= 72\left(\frac{5}{8}\right) + 83 = 45 + 83 = 128$$

Question58

Let $A(\alpha, -2)$, $B(\alpha, 6)$ and $C\left(\frac{\alpha}{4}, -2\right)$ be vertices of a $\triangle ABC$. If $\left(5, \frac{\alpha}{4}\right)$ is the circumcentre of $\triangle ABC$, then which of the following is NOT correct about $\triangle ABC$?

[29-Jul-2022-Shift-2]

Options:

A. area is 24

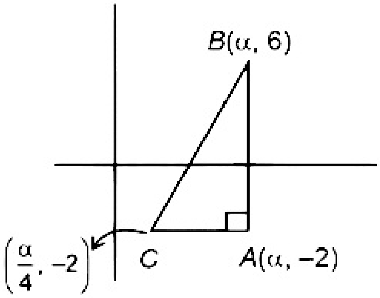
B. perimeter is 25

C. circumradius is 5

D. inradius is 2

Answer: B

Solution:



Circumcentre of △ABC

$= \left(\frac{\alpha + \frac{\alpha}{4}}{2}, \frac{6-2}{2} \right)$

$= \left(\frac{5\alpha}{8}, 2 \right)$

$= \left(5, \frac{\alpha}{4} \right)$

$\Rightarrow \alpha = 8$

area(△ABC) $= \frac{1}{2} \cdot \frac{3\alpha}{4} \times 8 = 24$ sq. units

Perimeter $= 8 + \frac{3\alpha}{4} + \sqrt{8^2 + \left(\frac{3\alpha}{4} \right)^2} = 8 + 6 + 10 = 24$

Circumradius $= \frac{10}{2} = 5$

$r = \frac{\Delta}{s} = \frac{24}{12} = 2$

Question59

A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two parts such that QR = 15m. If from a point A on the ground the angle of elevation of R is 60° and the part PR of the tower subtends an angle of 15° at A, then the height of the tower is : [25-Jul-2022-Shift-1]

Options:

A. $5(2\sqrt{3} + 3)$m

B. $5(\sqrt{3} + 3)$m

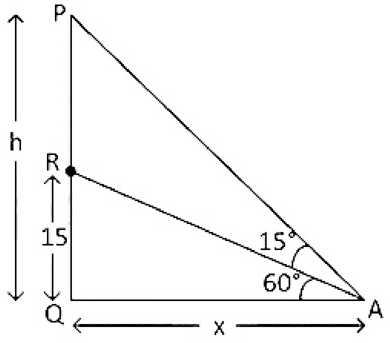
C. $10(\sqrt{3} + 1)$m

D. $10(2\sqrt{3} + 1)$m

Answer: A

Solution:

Solution:



For $\triangle AQR$,

$\tan 60^{\circ} = \frac{15}{x}$..... (1

From $\triangle AQP$,

$\tan 75^{\circ} = \frac{h}{x}$

$\Rightarrow (2+\sqrt{3}) = \frac{h}{x} [\because \tan 75^{\circ} = 2+\sqrt{3}]$

$\Rightarrow h = (2+\sqrt{3})x$

$= (2+\sqrt{3}) \frac{15}{\sqrt{3}} [\text{ From } (1)]$

$= (2+\sqrt{3}) \times \frac{15\sqrt{3}}{3}$

$= (2+\sqrt{3}) \times 5\sqrt{3}$

$= 5(2\sqrt{3}+3)m$

Question60

Let a vertical tower AB of height 2h stands on a horizontal ground. Let from a point P on the ground a man can see upto height h of the tower with an angle of elevation 2α. When from P, he moves a distance d in the direction of $\overrightarrow{AP}$, he can see the top B of the tower with an angle of elevation α. If $d = \sqrt{7}h$, then $\tan \alpha$ is equal to
[27-Jul-2022-Shift-1]

Options:

A. $\sqrt{5} - 2$

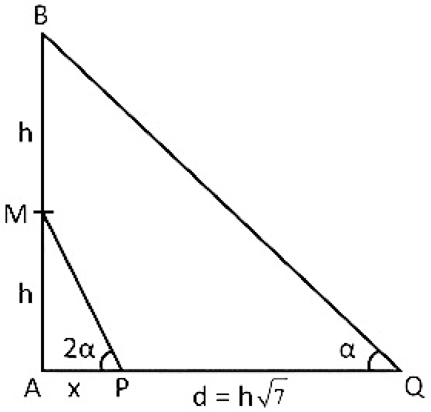
B. $\sqrt{3} - 1$

C. $\sqrt{7} - 2$

D. $\sqrt{7} - \sqrt{3}$

Answer: C

Solution:



$\triangle APM$ gives

$\tan 2\alpha = \frac{h}{x}$..... (i)

$\triangle AQB$ gives

$\tan 2\alpha = \frac{2h}{x+d} = \frac{2h}{x+h\sqrt{7}}$...... (ii)

From (i) and (ii)

$\tan 2\alpha = \frac{2\cdot\tan 2\alpha}{1+\sqrt{7}\cdot\tan 2\alpha}$

Let $t = \tan\alpha$

$\Rightarrow t = \frac{2\frac{2t}{1-t^2}}{1+\sqrt{7}\cdot\frac{2t}{1-t^2}}$

$\Rightarrow t^2 - 2\sqrt{7}t + 3 = 0$

$t = \sqrt{7} - 2$

Question61

The angle of elevation of the top P of a vertical tower PQ of height 10 from a point A on the horizontal ground is 45°. Let R be a point on AQ and from a point B, vertically above R, the angle of elevation of P is 60°. If ∠BAQ = 30°, AB = d and the area of the trapezium PQRB is α, then the ordered pair (d, α) is :
[27-Jul-2022-Shift-2]

Options:

A. $(10(\sqrt{3}-1), 25)$

B. $\left(10(\sqrt{3}-1), \frac{25}{2}\right)$

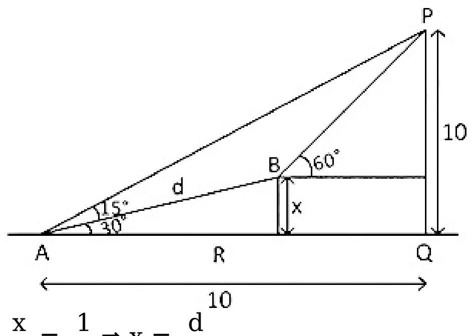
C. $(10(\sqrt{3}+1), 25)$

D. $\left(10(\sqrt{3}+1), \frac{25}{2}\right)$

Answer: A

Solution:

Solution:
Let BR = x



$\frac{x}{d} = \frac{1}{2} \Rightarrow x = \frac{d}{2}$

$\frac{10 - x}{10 - x\sqrt{3}} = \sqrt{3} \Rightarrow 10 - x = 10\sqrt{3} - 3x$

$2x = 10(\sqrt{3} - 1)$

$x = 5(\sqrt{3} - 1)$

$d = 2x = 10(\sqrt{3} - 1)$

$\alpha = \frac{1}{2}(x + 10)(10 - x\sqrt{3}) = \text{Area}(PQRB)$

$= \frac{1}{2}(5\sqrt{3} - 5 + 10)(10 - 5\sqrt{3}(\sqrt{3} - 1))$

$= \frac{1}{2}(5\sqrt{3} + 5)(10 - 15 + 5\sqrt{3}) - \frac{1}{2}(75 - 25) = 25$

Question62

A horizontal park is in the shape of a triangle OAB with AB = 16. A vertical lamp post OP is erected at the point O such that $\angle PAO = \angle PBO = 15^{\circ}$ and $\angle PCO = 45^{\circ}$, where C is the midpoint of AB. Then $(OP)^2$ is equal to
[28-Jul-2022-Shift-2]

Options:

A. $\frac{32}{\sqrt{3}}(\sqrt{3} - 1)$

B. $\frac{32}{\sqrt{3}}(2 - \sqrt{3})$

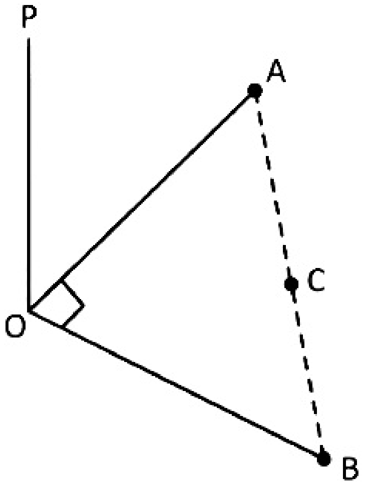
C. $\frac{16}{\sqrt{3}}(\sqrt{3} - 1)$

D. $\frac{16}{\sqrt{3}}(2 - \sqrt{3})$

Answer: B

Solution:

Solution:



$OP = OA\tan 15 = OB\tan 15$....... (i)
$OP = OC\tan 45 \Rightarrow OP = OC$....... (ii)
$OA = OB$...... (iii)
$OC^2 + 8^2 = OA^2$

$$OP^2 + 64 = OP^2\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2$$

$$64 = OP^2\left[\frac{(\sqrt{3}+1)^2 - (\sqrt{3-1})^2}{(\sqrt{3}-1)^2}\right]$$

$$= OP^2\left(\frac{4\sqrt{3}}{(\sqrt{3}-1)^2}\right)$$

$$OP^2 = \frac{64(\sqrt{3}-1)^2}{4\sqrt{3}} = \frac{32}{\sqrt{3}}(2-\sqrt{3})$$

Question63

The angle of elevation of the top of a tower from a point A due north of it is α and from a point B at a distance of 9 units due west of A is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. If the distance of the point B from the tower is 15 units, then $\cot\alpha$ is equal to :
[29-Jul-2022-Shift-1]

Options:

A. $\frac{6}{5}$

B. $\frac{9}{5}$

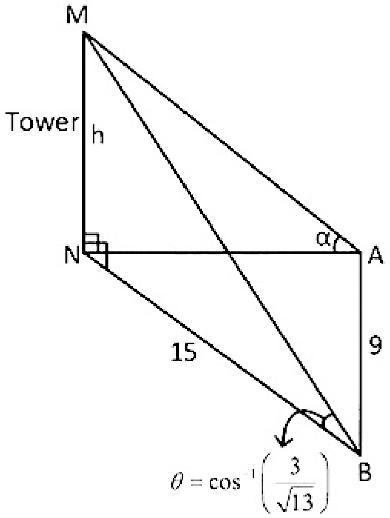
C. $\frac{4}{3}$

D. $\frac{7}{3}$

Answer: A

Solution:

Solution:



$NA = \sqrt{15^2 - 9^2} = 12$

$\frac{h}{15} = \tan\theta = \frac{2}{3}$

h = 10 units

$\cot\alpha = \frac{12}{10} = \frac{6}{5}$

Question64

The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a [2021, 26 Feb. Shift-1]

Options:

A. right angled triangle

B. equilateral triangle

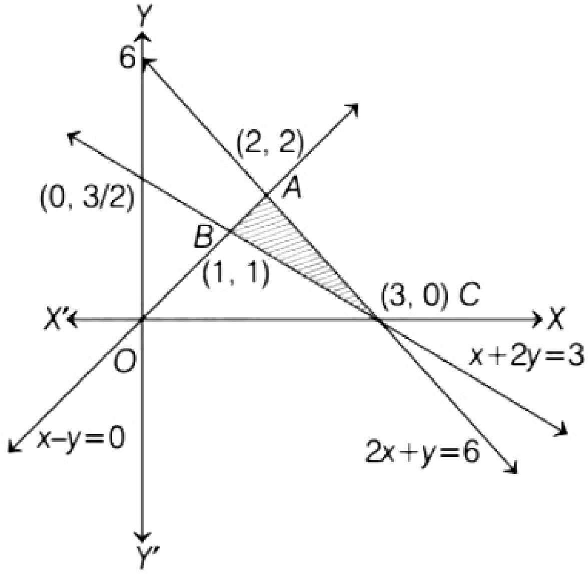
C. isosceles triangle

D. None of these

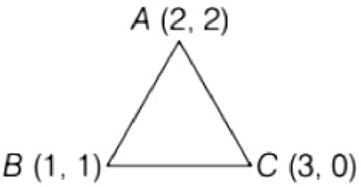
Answer: C

Solution:

Solution:
Given lines, $x - y = 0$, $x + 2y = 3$, $2x + y = 6$



The only triangle which include all three lines is $\triangle ABC$.



Now, $AB = \sqrt{(2-1)^2 + (2-1)^2} = \sqrt{2}$
$AC = \sqrt{(2-3)^2 + (2-0)^2} = \sqrt{5}$
$BC = \sqrt{(3-1)^2 + (0-1)^2} = \sqrt{5}$
$\Rightarrow$ AC = BC (two sides are equal)
$\Rightarrow \Delta$ ABC is isosceles triangle.

Question65

If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is [2021, 25 Feb. Shift-II]

Options:

A. $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

B. $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$

C. $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$

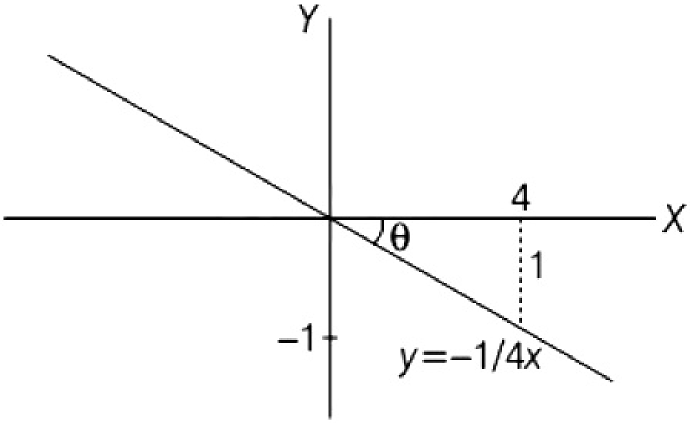
D. $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$

Answer: A

Solution:

Solution:
Curve $x^2 + 2y^2 = 2$ intersect the line $x + y = 1$ at points P and Q . First we have to find any common relation between these two curves.
Use substitution for the same as follows,

$x^2 + 2y^2 = 2 \quad \cdots\cdots (i)$
$x + y = 1$, then $(x + y)^2 = 1^2$
$\Rightarrow x^2 + y^2 + 2xy = 1 \quad \cdots\cdots (ii)$
We can write Eq. (i) as,
$x^2 + 2y^2 - 2(1)^2 = 0$
$\Rightarrow x^2 + 2y^2 - 2(x + y)^2 = 0$ [using Eq. (ii) in Eq. (i)]
$\Rightarrow x^2 + 2y^2 - 2x^2 - 2y^2 - 4xy = 0$
$\Rightarrow -x^2 - 4xy = 0 \Rightarrow -x(x + 4y) = 0$
Given, $x = 0$ and $x + 4y = 0$ or $y = \frac{-1}{4}x$
Draw the line $y = \frac{-1}{4}x$ on graph and take
arbitrary point (any one) as follows,
From given graph,



$\tan\theta = \frac{1}{4} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{4}\right)$
We have two lines, $y = -\frac{1}{4}x$ and $x = 0$ (i.e. Y-axis).
Thus, any line joining these two curves makes an angle $\frac{\pi}{2} + \theta$ at origin.
$\therefore$ Answer is $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$.

Question66

The image of the point (3, 5) in the line $x - y + 1 = 0$, lies on [2021, 25 Feb. Shift-1]

Options:

A. $(x - 2)^2 + (y - 2)^2 = 12$

B. $(x - 4)^2 + (y + 2)^2 = 16$

C. $(x - 4)^2 + (y - 4)^2 = 8$

D. $(x - 2)^2 + (y - 4)^2 = 4$

Answer: D

Solution:

Solution:
Image of $P(3, 5)$ on the line $x - y + 1 = 0$ is
$$\frac{x-3}{1} = \frac{y-5}{-1} = \frac{-2(3-5+1)}{2}$$

$\Rightarrow \dfrac{x-3}{1} = \dfrac{y-5}{-1} = 1$

$\Rightarrow \dfrac{x-3}{1} = 1$ and $\dfrac{y-5}{-1} = 1$

$x = 4, y = 4$

$\therefore$ Required image is at $(4, 4)$.

Clearly, this point lies on

$(x-2)^2 + (y-4)^2 = 4$ as

$(4, 4)$ satisfies this equation.

Question67

A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is\/are on the path of the man? 24 Feb 2021 Shift 1

Options:

A. A only

B. C only

C. All the three

D. B only

Answer: D

Solution:

Solution:

Let the line be $y = mx + c$

$\therefore$ x-intercept : $-\dfrac{c}{m}$

y-intercept : c

A.M. of reciprocals of the intercepts :

$$\frac{-\frac{m}{c} + \frac{1}{c}}{2} = \frac{1}{4} \Rightarrow 2(1-m) = c$$

Line : $y = mx + 2(1-m) = c$

$\Rightarrow (y-2) - m(x-2) = 0$

$\Rightarrow$line always passes through $(2, 2)$

Question68

Let $A(-1, 1)$, $B(3, 4)$ and $C(2, 0)$ be given three points. A line $y = mx$, $m > 0$ intersects lines AC and BC at point P and Q_1 respectively. Let A_1 and A_2 be the areas of $\triangle ABC$ and $\triangle PQC_1$ respectively, such that $A_1 = 3A_2$, then the value of m is equal to [2021, 16 March Shift-II]

Options:

A. $\frac{4}{15}$

B. 1

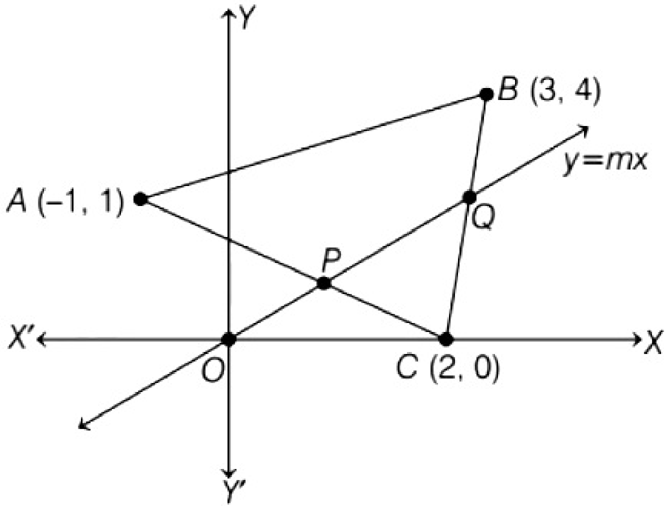
C. 2

D. 3

Answer: B

Solution:

Solution:
Given, points $A(-1, 1)$, $B(3, 4)$, $C(2, 0)$
Equation of AC = $\frac{y-1}{x+1} = \frac{0-1}{2+1} = \frac{-1}{3}$
$\Rightarrow 3y - 3 = -x - 1 \Rightarrow x + 3y = 2 \quad \cdots\cdots (i)$
On solving Eq. (i) and $y = mx$, we get
$P\left(\frac{2}{3m+1}, \frac{2m}{3m+1}\right)$
Equation of BC = $\frac{y-0}{x-2} = \frac{4-0}{3-2}$
$\Rightarrow \; y = 4x - 8 \quad \cdots\cdots (ii)$
Similarly, on solving Eq. (ii) and $y = mx$,
we get $0\left(\frac{8}{4-m}, \frac{8m}{4-m}\right)$



Area of $\triangle ABC$ = 3 Area of $\triangle PQC$ (given)

$$\frac{1}{2}\begin{vmatrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 3 & 4 & 1 \end{vmatrix} = 3 \times \frac{1}{2}\begin{vmatrix} 2 & 0 & 1 \\ \frac{8}{4-m} & \frac{8m}{2} & 1 \\ \frac{2}{3m+1} & \frac{2m}{3m+1} & 1 \end{vmatrix}$$

$$\Rightarrow 13 = 3\left(\frac{1}{4-m}\right)\left(\frac{1}{3m+1}\right)\begin{vmatrix} 2 & 0 & 1 \\ 8 & 8m & 4-m \\ 2 & 2m & 3m+1 \end{vmatrix}$$

$\Rightarrow 13 = \frac{3}{4 + 11m - 3m^2} \times (52m^2)$
$\Rightarrow 15m^2 - 11m - 4 = 0$
$\Rightarrow m = 1, \frac{-4}{15}$ [but $m > 0$]
$\Rightarrow m = 1$

Question69

In a $\triangle PQR$, the coordinates of the points P and Q are $(-2, 4)$ and $(4, -2)$, respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the $\triangle PQR$ is [2021, 17 March Shift-1]

Options:

A. $(-1, 0)$

B. $(-2, -2)$

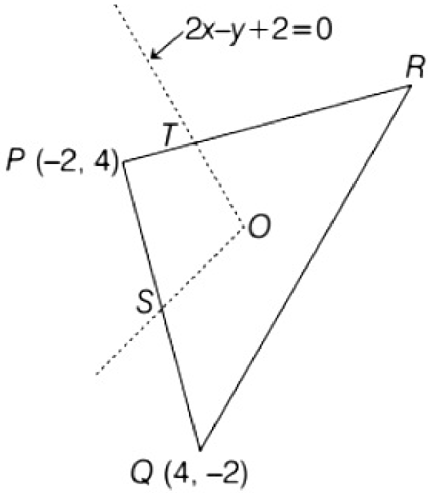
C. $(0, 2)$

D. $(1, 4)$

Answer: B

Solution:

Solution:
Let O be the centre of the circumcircle.



And T be the mid-point of PR .
So, equation of OT is given as
$2x - y + 2 = 0$
Let S be the mid-point of PQ .
Now, S will be
$\left(\frac{-2+4}{2}, \frac{4-2}{2}\right) = (1, 1)$
Equation of OS = $\frac{y-1}{x-1} = \frac{-1}{m_{PQ}}$
$m_{PQ} = \frac{-2-4}{4+2} = -1$
$\therefore$ OS = $y - 1 = 1(x - 1)$
$y = x$
Now, coordinates of O will be the intersection of lines OS and OT.
$$\begin{cases} y = x \\ 2x - y + 2 = 0. \end{cases}$$
$\Rightarrow 2x - x + 2 = 0 \Rightarrow x = -2$
$\therefore$ $y = -2 \Rightarrow 0 = (-2, -2)$

Question70

The number of integral values of m, so that the abscissa of point of

intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is [2021, 18 March Shift-1]

Options:

A. 1

B. 2

C. 3

D. 0

Answer: B

Solution:

Solution:
Given, $y = mx + 1$
and $3x + 4y = 9$
From Eqs. (i) and (ii),
$3x + 4(mx + 1) = 9$
$\Rightarrow 3x + 4mx + 4 = 9$
$\Rightarrow x(3 + 4m) = 5$
$\Rightarrow x = \dfrac{5}{3 + 4m}$
Given, that the abscissa of point of intersection of Eqs. (i) and (ii) i.e. $x = \dfrac{5}{3 + 4m}$ is an integer.
∴ Possible values of x are
$x = 1, -1, 5, -5$
i.e. $\dfrac{5}{4m + 3} = 1$ or $\dfrac{5}{4m + 3} = -1$
or $\dfrac{5}{4m + 3} = 5$ or $\dfrac{5}{4m + 3} = -5$
$\Rightarrow 4m = 2$ or $-4m = 8$
or $4m = -2$ or $-4m = 4$
$\Rightarrow m = \dfrac{1}{2}, -2, -\dfrac{1}{2}, -1$
$\because \left\{ -\dfrac{1}{2}, \dfrac{1}{2} \right\} \notin 1$
$\therefore m = \{-1, -2\} \in 1$
∴ Number of integral values of mare 2 .

Question71

The equation of one of the straight lines which passes through the point (1, 3) and makes an angle $\tan^{-1}(\sqrt{2})$ with the straight line, $y + 1 = 3\sqrt{2}x$ is
[2021, 18 March Shift-1]

Options:

A. $4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$

B. $5\sqrt{2}x + 4y - (15 + 4\sqrt{2}) = 0$

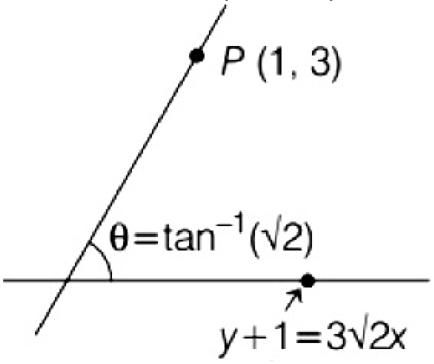
C. $4\sqrt{2}x + 5y - 4\sqrt{2} = 0$

D. $4\sqrt{2}x - 5y - (5 + 4\sqrt{2}) = 0$

Answer: A

Solution:

Solution:
Method 1 Let $m =$ Slope of required line



$\therefore$ Equation of required line
$y - 3 = m(x - 1)$
Given, equation of line is
$3\sqrt{2}x - y - 1 = 0$
Since, angle θ between Eqs. (i) and (ii) is $\tan^{-1}(\sqrt{2})$
i.e. $\tan\theta = \sqrt{2}$
$\Rightarrow \left|\frac{m - 3\sqrt{2}}{1 + 3\sqrt{2}m}\right| = \sqrt{2}$
($\because$ Slope of Eq. (i) = mand slope of Eq. (ii) $= 3\sqrt{2}$)
Squaring on both sides,
$m^2 - 6\sqrt{2}m + 18 = 2(1 + 18m^2 + 6\sqrt{2}m)$
$\Rightarrow 35m^2 + 18\sqrt{2}m - 16 = 0$
$\therefore\ m = \frac{-18\sqrt{2} \pm \sqrt{648 + 2240}}{70}$
$= \frac{-18\sqrt{2} \pm 38\sqrt{2}}{70}$
$\Rightarrow m = \frac{2\sqrt{2}}{7}, -\frac{4}{5}\sqrt{2}$
For $m = \frac{2\sqrt{2}}{7}$, equation of required line
will be
$y - 3 = \frac{2\sqrt{2}}{7}(x - 1)$
$\Rightarrow 2\sqrt{2}x - 7y + 21 - 2\sqrt{2} = 0$
(options are not matching so, neglect this)
For $m = \frac{-4\sqrt{2}}{5}$, equation of required line
will be
$y - 3 = \frac{-4\sqrt{2}}{5}(x - 1)$
$\Rightarrow 5y - 15 = -4\sqrt{2}x + 4\sqrt{2}$
$\Rightarrow 4\sqrt{2}x + 5y - 15 - 4\sqrt{2} = 0$
$\Rightarrow 4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$

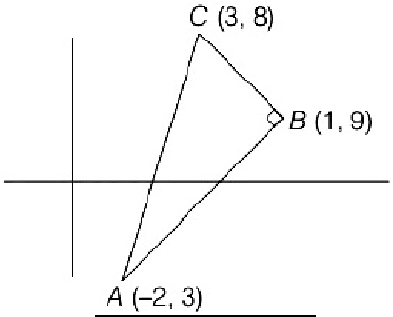
Question72

Consider a triangle having vertices $A(-2, 3)$, $B(1, 9)$ and $C(3, 8)$. If a line L passing through the circumcentre of $\triangle ABC$, bisects line BC, and intersects Y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is [2021, 20 July Shift-II]

Answer: 9

Solution:

Solution:



$AB = \sqrt{(1+2)^2 + (9-3)^2} = \sqrt{45}$
$BC = \sqrt{(3-1)^2 + (8-9)^2} = \sqrt{5}$
$AC = \sqrt{(3+2)^2 + (8-3)^2} = \sqrt{50}$
$\therefore (\sqrt{50})^2 = (\sqrt{45})^2 + (\sqrt{5})^2$
$(AC)^2 = (AB)^2 + (BC)^2$
$\angle B = 90^\circ$
$\Rightarrow$ABC is right angled triangle.
Circumcentre = Mid-point of hypotenuse
= Mid-point of AC
$= \left(\frac{1}{2}, \frac{11}{2}\right)$
Mid-point of line BC = $\left(2, \frac{17}{2}\right)$
Line passing through circumcentre and bisect line BC will be

$$y - \frac{11}{2} = \frac{\frac{17}{2} - \frac{11}{2}}{2 - \frac{1}{2}}\left(x - \frac{1}{2}\right)$$

$$\Rightarrow y - \frac{11}{2} = \frac{3 \times 2}{3}\left(x - \frac{1}{2}\right)$$

$$\Rightarrow y - \frac{11}{2} = 2\left(x - \frac{1}{2}\right)$$

It passes through $\left(0, \frac{\alpha}{2}\right)$.

$$\therefore \frac{\alpha}{2} - \frac{11}{2} = 2\left(0 - \frac{1}{2}\right) \Rightarrow \alpha - 11 = 4\left(-\frac{1}{2}\right)$$

$\Rightarrow \alpha = 11 - 2 = 9$
$\Rightarrow \alpha = 9$

Question73

Let the equation of the pair of lines, y = px and y = qx can be written as (y − px)(y − qx) = 0 Then, the equation of the pair of the angle bisectors of the line $x^2 - 4xy - 5y^2 = 0$ is
[2021, 25 July Shift-II]

Options:

A. $x^2 - 3xy + y^2 = 0$

B. $x^2 + 4xy - y^2 = 0$

C. $x^2 + 3xy - y^2 = 0$

D. $x^2 - 3xy - y^2 = 0$

Answer: C

Solution:

Solution:
Equation of angle bisector of homogeneous equation of pair of straight line $ax^2 + 2hxy + by^2$ is
$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$
For $x^2 - 4xy - 5y^2 = 0$
$a = 1, h = -2, b = -5$
So, equation of angle bisector is
$$\frac{x^2 - y^2}{1 - (-5)} = \frac{xy}{-2} \Rightarrow x^2 - y^2 = -3xy \Rightarrow x^2 + 3xy - y^2 = 0$$
So, combined equation of angle bisector is
$x^2 + 3xy - y^2 = 0$. is $x^2 + 3xy - y^2 = 0$.

Question74

Two sides of a parallelogram are along the lines $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals of the parallelogram is $11x + 7y = 9$, then other diagonal passes through the point [2021, 27 July Shift-II]

Options:

A. (1,2)

B. (2,2)

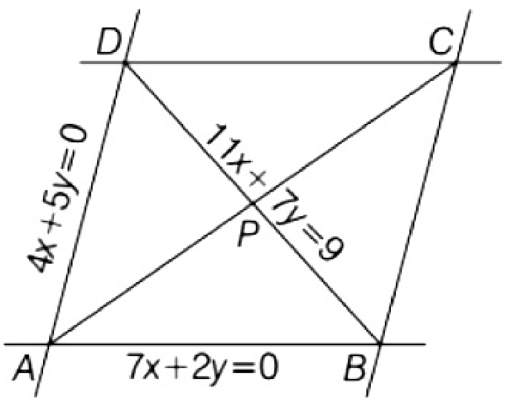
C. (2,1)

D. (1,3)

Answer: B

Solution:

Solution:
Given, two sides of parallelogram are
$r4x + 5y = 0$
$7x + 2y = 0$
and $7x + 2y = 0$



Both lines are passing through origin.
Thus, point $A = (0, 0)$

The equation of diagonal is $11x + 7y = 9$
Point D is the point of intersection of
$4x + 5y = 0$
and $11x + 7y = 9$
So, coordinate of $D = \left(\frac{5}{3}, -\frac{4}{3}\right)$
Also, point B is the point of intersection of $7x + 2y = 0$ and $11x + 7y = 9$
So, coordinate of point $B = \left(-\frac{2}{3}, \frac{7}{3}\right)$
We know that, diagonals of parallelogram bisects each other. Let P is the middle point of BD.

So, coordinate of
$$P = \left(\frac{\frac{5}{3} + \left(-\frac{2}{3}\right)}{2}, \frac{-\frac{4}{3} + \frac{7}{3}}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$$
Now, equation of diagonal AC
$$y - 0 = \frac{\frac{1}{2} - 0}{\frac{1}{2} - 0}(x - 0)$$
$$\Rightarrow y = \frac{\frac{1}{2}}{\frac{1}{2}}x \Rightarrow y = x$$
$\therefore$ Diagonal AC passes through (2, 2).

Question75

Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the Y-axis at C. The locus of the mid-point P of MC is [2021, 27 Aug. Shift-1]

Options:

A. $3x^2 - 2y - 6 = 0$

B. $3x^2 + 2y - 6 = 0$

C. $2x^2 + 3y - 9 = 0$

D. $2x^2 - 3y + 9 = 0$

Answer: C

Solution:

Solution:
Given, A(0, 6) and B(2t, 0)
Mid-point of AB = M (t, 3)
Equation of perpendicular bisector of
AB passes through M .
$\therefore\ y - 3 = \frac{t}{3}(x - t) \quad \cdots\cdots (i)$
So, $C\left(0, 3 - \frac{t^2}{3}\right)$
Intersection of Eq. (i) on Y -axis
$C\left(0, 3 - \frac{t^2}{3}\right)$

Let mid-point of M C is (h, k) .

Then, $(h, k) = \left(\frac{t}{2}, 3 - \frac{t^2}{6}\right)$

$\Rightarrow h = \frac{t}{2}, k = 3 - \frac{t^2}{6}$

Eliminatingt, we get

$2h^2 = 3(3 - k)$

Locus of (h, k)

$2x^2 = 3(3 - y)$

$\Rightarrow 2x^2 + 3y - 9 = 0$

Question76

Let ABC be a triangle with A(−3, 1) and ∠ACB = θ, 0 < θ < $\frac{\pi}{2}$. If the equation of the median through B is 2x + y − 3 = 0 and the equation of angle bisector of C is 7x − 4y − 1 = 0, then tan θ is equal to [2021, 26 Aug. Shift-I]

Options:

A. 1 / 2

B. 3 / 4

C. 4/3

D. 2

Answer: C

Solution:

Solution:

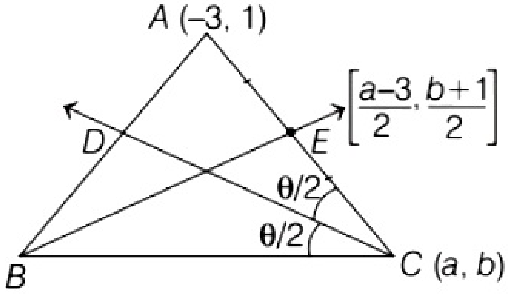
Given, the equation of median through B

i.e. BE : $2x + y - 3 = 0$

Equation, of angle bisector of C i.e.

CD : $7x - 4y = 1$

Since, E satisfies the equation of BE .



$r2\left(\frac{a-3}{2}\right) + \left(\frac{b+1}{2}\right) - 3 = 0$

$2a - 6 + b + 1 - 6 = 0$

$2a + b = 11 \quad \cdots\cdots (i)$

Since, C satisfies C(d)

$\therefore 7a - 4b = 1 \quad \cdots\cdots (ii)$

On solving Eqs. (i) and (ii), we get

$a = 3, b = 5$

Slope of AC = $\frac{2}{3}$ Slope of CD = $\frac{7}{4}$

$$\therefore \tan\left(\frac{\theta}{2}\right) = \left| \frac{\frac{2}{3} - \frac{7}{4}}{1 + \frac{14}{12}|} \right| = \frac{1}{2}$$

$$\text{Now, } \tan\theta = \frac{2\tan\left(\frac{\theta}{2}\right)}{1 - \tan^2\left(\frac{\theta}{2}\right)} = \frac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} = \frac{4}{3}$$

Question77

If p and q are the lengths of the perpendiculars from the origin on the lines,
$x\operatorname{cosec}\alpha - y\sec\alpha = k\cot 2\alpha$ and $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$
respectively, then k^2 is equal to
[2021, 31 Aug. Shift-1]

Options:

A. $4p^2 + q^2$

B. $2p^2 + q^2$

C. $p^2 + 2q^2$

D. $2p^2 + q^2\ p^2 + 4q^2$

Answer: A

Solution:

Solution:

$$p = \frac{k\cot 2\alpha}{\sqrt{\operatorname{cosec}^2\alpha + \sec^2\alpha}}$$

$$\Rightarrow\ q = \frac{k\sin 2\alpha}{\sqrt{\sin^2\alpha + \cos^2\alpha}}$$

$$\Rightarrow p = \frac{k\left(\frac{\cos 2\alpha}{\sin 2\alpha}\right)}{\sqrt{\frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha\cos^2\alpha}}} = \frac{k\cos 2\alpha}{\sin 2\alpha}$$

$$\Rightarrow\ p = \left(\frac{k}{2}\right)\cos 2\alpha$$

$\Rightarrow\ q = k\sin 2\alpha$

$\Rightarrow \cos 2\alpha = (2p / k)$

$\Rightarrow\ \sin 2\alpha = (q / k)$

$\Rightarrow \sin^2 2\alpha + \cos^2 2\alpha = 1$

$$\Rightarrow\ \frac{4p^2}{k^2} + \frac{q^2}{k^2} = 1$$

$\Rightarrow\ 4p^2 + q^2 = k^2$

Question78

Let A(1, 0), B(6, 2) and C$\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is

a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is ________.
[NA Jan. 7, 2020 (I)]

Answer: 5

Solution:

Solution:
P will be centroid of $\triangle ABC$
$P\left(\frac{17}{6}, \frac{8}{3}\right) \Rightarrow PQ = \sqrt{(4)^2 + (3)^2} = 5$

Question79

The locus of the mid-points of the perpendiculars drawn from points on the line, x = 2y to the line x = y is:
[Jan. 7, 2020 (II)]

Options:

A. $2x - 3y = 0$

B. $5x - 7y = 0$

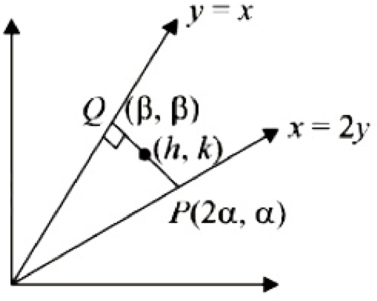
C. $3x - 2y = 0$

D. $7x - 5y = 0$

Answer: B

Solution:

Solution:



Since, slope of $PQ = \frac{k - \alpha}{h - 2\alpha} = -1$
$\Rightarrow k - \alpha = -h + 2\alpha$
$\Rightarrow \alpha = \frac{h + k}{3}$
Also, $2h = 2\alpha + \beta$ and
$2k = \alpha + \beta$
$\Rightarrow 2h = \alpha + 2k$

$\Rightarrow \alpha = 2h - 2k$
From (i) and (ii), we have
$\frac{h+k}{3} = 2(h-k)$
So, locus is $6x - 6y = x + y$
$\Rightarrow 5x = 7y \Rightarrow 5x - 7y = 0$

Question80

Let C be the centroid of the triangle with vertices (3,-1) (1,3) and (2,4) . Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$. Then the line passing through the points C and P also passes through the point: [Jan. 9, 2020 (I)]

Options:

A. (-9,-6)

B. (9,7)

C. (7,6)

D. (-9,-7)

Answer: A

Solution:

Solution:
Coordinates of centroides
$C = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$
$= \left(\frac{3+1+2}{3}, \frac{-1+3+4}{3}\right) = (2, 2)$
The given equation of lines are
$x + 3y - 1 = 0$ . . . (i)
$3x - y + 1 = 0$ . . . (ii)
Then, from (i) and (ii)
point of intersection $P\left(-\frac{1}{5}, \frac{2}{5}\right)$
equation of line DP
$8x - 11y + 6 = 0$

Question81

If a ΔABC has vertices $A(-1, 7)$, $B(-7, 1)$ and $C(5, -5)$, then its orthocentre has coordinates: [Sep. 03, 2020 (II)]

Options:

A. $\left(-\frac{3}{5}, \frac{3}{5}\right)$

B. (-3,3)

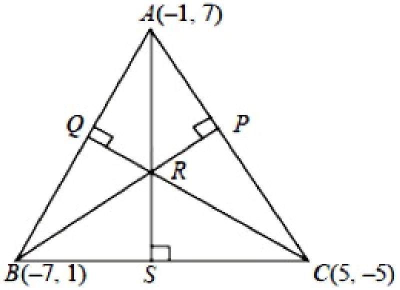
C. $\left(\frac{3}{5}, -\frac{3}{5} \right)$

D. (3,-3)

Answer: B

Solution:

Solution:



$m_{BC} = \frac{6}{-12} = -\frac{1}{2}$

∴ Equation of AS is $y - 7 = 2(x + 1)$

$y = 2x + 9$ . . . (i)

$m_{AC} = \frac{12}{-6} = -2$

∴ Equation of BP is $y - 1 = \frac{1}{2}(x + 7)$

$y = \frac{x}{2} + \frac{9}{2}$ . . . (ii)

From equs. (i) and (ii),

$2x + 9 = \frac{x + 9}{2}$

$\Rightarrow 4x + 18 = x + 9$

$\Rightarrow 3x = 9 \Rightarrow x = -3$

$\therefore y = 3$

Question82

A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90^\circ$, andar $(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is: [Sep. 04, 2020 (I)]

Options:

A. $1 + \sqrt{5}$

B. $1 + 2\sqrt{5}$

C. $2 + \sqrt{5}$

D. $2\sqrt{5} - 1\dot{m}$

Answer: B

Solution:

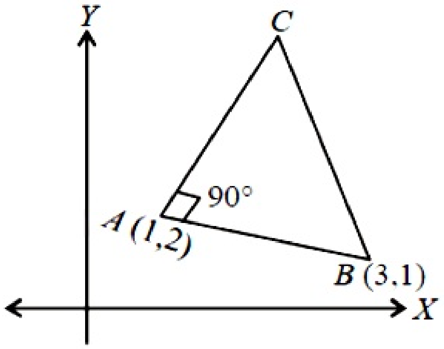
Solution:

Let ΔABC be in the first quadrant

Slope of line $AB = -\frac{1}{2}$

Slope of line $AC = 2$

Length of $AB = \sqrt{5}$



It is given that ar $(\Delta ABC) = 5\sqrt{5}$

$\therefore \frac{1}{2}AB \cdot AC = 5\sqrt{5} \Rightarrow AC = 10$

$\therefore$ Coordinate of vertex C $= (1 + 10\cos\theta, 2 + 10\sin\theta)$

$\because \tan\theta = 2 \Rightarrow \cos\theta = \frac{1}{\sqrt{5}}, \sin\theta = \frac{2}{\sqrt{5}}$

$\therefore$ Coordinate of C $= (1 + 2\sqrt{5}, 2 + 4\sqrt{5})$

$\therefore$ Abscissa of vertex C is $1 + 2\sqrt{5}$.

Question83

If the perpendicular bisector of the line segment joining the points P(1, 4) and Q(k, 3) has y -intercept equal to -4 then a value of k is : [Sep. 04, 2020 (II)]

Options:

A. -2

B. -4

C. $\sqrt{14}$

D. $\sqrt{15}$

Answer: B

Solution:

Solution:

Mid point of line segment PQ be $\left(\frac{k+1}{2}, \frac{7}{2}\right)$.

$\therefore$ Slope of perpendicular line passing through

(0,-4) and $\left(\frac{k+1}{2}, \frac{7}{2}\right) = \dfrac{\frac{7}{2} + 4}{\frac{k+1}{2} - 0} = \frac{15}{k+1}$

Slope of PQ $= \frac{4-3}{1-k} = \frac{1}{1-k}$

$\therefore \frac{15}{1+k} \times \frac{1}{1-k} = -1$

$1 - k^2 = -15 \Rightarrow k = \pm 4$

Question84

If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible value of α and β is _______.
[NA Sep. 05, 2020 (I)]

Answer: 30

Solution:

$L_1 : 2x - y + 3 = 0$

$L_1 : 4x - 2y + \alpha = 0 \Rightarrow 2x - y + \frac{\alpha}{2} = 0$

$L_1 : 6x - 3y + \beta = 0 \Rightarrow 2x - y + \frac{\beta}{3} = 0$

Distance between L_1 and L_2

$$\left| \frac{\alpha - 6}{2\sqrt{5}} \right| = \frac{1}{\sqrt{5}} \Rightarrow \left| \alpha - 6 \right| = 2$$

$\Rightarrow \alpha = 4, 8$

Distance between L_1 and L_3 :

$$\left| \frac{\beta - 9}{3\sqrt{5}} \right| = \frac{2}{\sqrt{5}} \Rightarrow \left| \beta - 9 \right| = 6$$

$\Rightarrow \beta = 15, 3$

Sum of all values $= 4 + 8 + 15 + 3 = 30$

Question85

Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2 & x < 0 \\ 0 & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2 & x > 0. \end{cases}$$

The value of lambda for which $f''(0)$ exists, is ______.
[NA Sep. 06, 2020 (I)]

Answer: 5

Solution:

$$f'(x) = \begin{cases} 5x^4 \cdot \sin\left(\frac{1}{x}\right) - x^3 \cos\left(\frac{1}{x}\right) + 10x & x < 0 \\ 0 & x = 0 \\ 5x^4 \cos\left(\frac{1}{x}\right) + x^3 \sin\left(\frac{1}{x}\right) + 2\lambda x & x > 0. \end{cases}$$

$$f''(x) = \begin{cases} (20x^3 - x)\sin\left(\frac{1}{x}\right) - 8x^2 \cos\left(\frac{1}{x}\right) + 10 & x < 0 \\ 0 & x = 0 \\ (20x^3 - x)\cos\left(\frac{1}{x}\right) + 8x^2 \sin\left(\frac{1}{x}\right) + 2\lambda & x > 0. \end{cases}$$

Now, $f''(0^+) = f''(0^-) \Rightarrow 2\lambda = 10 \Rightarrow \lambda = 5$

Question86

Let L denote the line in the xy -plane with x and y intercepts as 3 and 1 respectively. Then the image of the point (-1,-4) in this line is: [Sep. 06, 2020 (II)]

Options:

A. $\left(\frac{11}{5}, \frac{28}{5}\right)$

B. $\left(\frac{29}{5}, \frac{8}{5}\right)$

C. $\left(\frac{8}{5}, \frac{29}{5}\right)$

D. $\left(\frac{29}{5}, \frac{11}{5}\right)$

Answer: A

Solution:

Solution:
The line in xy -plane is,

$\frac{x}{3} + y = 1 \Rightarrow x + 3y - 3 = 0$

Let image of the point (-1,-4) be (α, β), then

$\frac{\alpha + 1}{1} = \frac{\beta + y}{3} = -\frac{2(-1 - 12 - 3)}{10}$

$\Rightarrow \alpha + 1 = \frac{\beta + 4}{3} = \frac{16}{5}$

$\Rightarrow \alpha = \frac{11}{5}, \beta = \frac{28}{5}$

Question87

Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true? [Jan. 9, 2019 (I)]

Options:

A. The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

B. Each line passes through the origin.

C. The lines are all parallel.

D. The lines are not concurrent.

Answer: A

Solution:

Solution:
The given equations of the set of all lines
$px + qy + r = 0 \ . \ . \ . \ (i)$
and given condition is :
$3p + 2q + 4r = 0$
$\Rightarrow \frac{3}{4}p + \frac{2}{4}q + r = 0 \ . \ . \ . \ (ii)$
From (i) & (ii) we get :
$\therefore \ x = \frac{3}{4}, y = \frac{1}{2}$
Hence the set of lines are concurrent and passing through the fixed point
$\left(\frac{3}{4}, \frac{1}{2}\right)$

Question88

Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1, 1), then the equation of its third side is:
[Jan. 09, 2019 (II)]

Options:

A. $122y - 26x - 1675 = 0$

B. $122y + 26x + 1675 = 0$

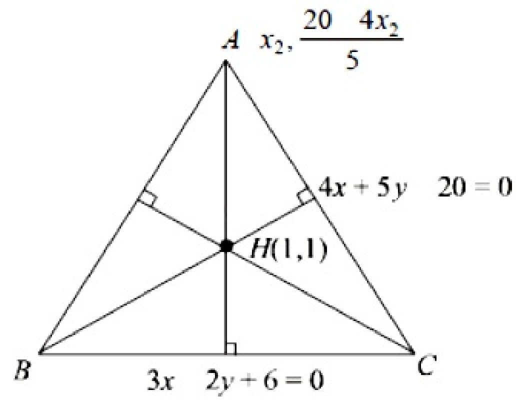
C. $26x + 61y + 1675 = 0$

D. $26x - 122y - 1675 = 0$

Answer: D

Solution:

Solution:



$\left(x_1, \ \frac{3x_1 + 6}{2}\right)$

Since, AH is perpendicular to BC

Hence, $m_{AH} \cdot m_{BC} = -1$

$$\left(\frac{\frac{20-4x_2}{5} - 1}{x_2 - 1} \right) \times \frac{3}{2} = -1$$

$$\frac{15 - 4x_2}{5(x_2 - 1)} = -\frac{2}{3}$$

$45 - 12x_2 = -10x_2 + 10$

$2x_2 = 35 \Rightarrow x_2 = \frac{35}{2}$

$\Rightarrow A\left(\frac{35}{2}, -10\right)$

Since, BH is perpendicular to CA.

Hence, $m_{BH} \times m_{CA} = -1$

$$\left(\frac{\frac{3x_1}{2} + 3 - 1}{x_1 - 1} \right) \left(-\frac{4}{5}\right) = -1$$

$$\frac{(3x_1 + 4)}{2(x_1 - 1)} \times 4 = 5$$

$\Rightarrow 6x_1 + 8 = 5x_1 - 5 \Rightarrow x_1 = -13 \Rightarrow \left(-13, \ \frac{-33}{2}\right)$

⇒ Equation of line AB is

$$y + 10 = \left(\frac{-\frac{33}{2} + 10}{-13 - 35} \right) \left(x - \frac{35}{2}\right)$$

$\Rightarrow -61y - 610 = -13x + \frac{455}{2}$

$\Rightarrow -122y - 1220 = -26x + 455$

$\Rightarrow 26x - 122y - 1675 = 0$

Question89

A point P moves on the line $2x - 3y + 4 = 0$. If $Q(1, 4)$ and $R(3, -2)$ are fixed points, then the locus of the centroid of ΔPQR is a line: [Jan. 10, 2019 (I)]

Options:

A. with slope $\frac{3}{2}$

B. parallel to x -axis

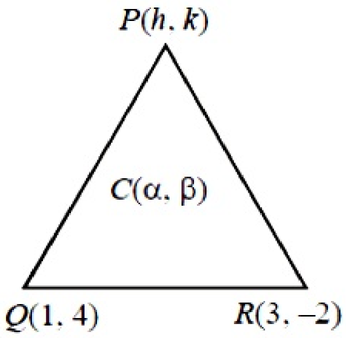
C. with slope $\frac{2}{3}$

D. parallel to y -axis

Answer: C

Solution:

Solution:



Let centroid C be (α, β)

we have $\alpha = \frac{1+3+h}{3} \Rightarrow h = 3\alpha - 4$

$\beta = \frac{4-2+k}{3} \Rightarrow k = 3\beta - 2$

but $P(h, k)$ lies on $2x - 3y + 4 = 0$

$\Rightarrow 2(3\alpha - 4) - 3(3\beta - 2) + 4 = 0$

$\Rightarrow 6\alpha - 9\beta - 8 + 6 + 4 = 0$

$\Rightarrow 6\alpha - 9\beta + 2 = 0$

Locus: $6x - 9y + 2 = 0$

$\Rightarrow y = \frac{6}{9}x + \frac{2}{9} \therefore$ its slope $= \frac{6}{9} = \frac{2}{3}$

Question90

If the line 3x + 4y − 24 = 0 intersects the x -axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is: [Jan. 10, 2019 (I)]

Options:

A. (3,4)

B. (2,2)

C. (4,3)

D. (4,4)

Answer: B

Solution:

Solution:

Equation of the line is: $3x + 4y = 24$ or $\frac{x}{8} + \frac{y}{6} = 1$

$\therefore$ coordinates of A, B & O are (8, 0), (0, 6) & (0, 0) respectively.

$\Rightarrow OA = 8, OB = 6$ & $AB = 10$

$\therefore$ Incentre of ΔOAB is given as:

$$I \equiv \left(\frac{8 \times 0 + 6 \times 8 + 10 \times 0}{8 + 6 + 10}, \frac{8 \times 6 + 6 \times 0 + 10 \times 0}{8 + 6 + 10} \right) \equiv (2, 2).$$

Question91

Two vertices of a triangle are (0,2) and (4,3) . If its orthocentre is at the origin, then its third vertex lies in which quadrant?
[Jan. 10, 2019 (II)]

Options:

A. third

B. second

C. first

D. fourth

Answer: B

Solution:

Solution:
Since, $m_{QR} \times m_{PH} = -1$
$\Rightarrow m_{QR} = -\frac{1}{m_{PH}}$
$\Rightarrow m_{QR} = \frac{y-3}{x-4} = 0$
$\Rightarrow\ y = 3$
$m_{PQ} \times m_{RH} = -1$
$\Rightarrow \frac{1}{4} \times \frac{y}{x} = -1$
$\Rightarrow y = -4x$
$\Rightarrow x = -\frac{3}{4}$
Vertex R is $\left(\frac{-3}{4}, 3\right)$
Hence, vertex R lies in second quadrant.

Question92

Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at $(2, 4)$, then one of its vertex is:
[Jan. 10, 2019 (II)]

Options:

A. (3,5)

B. (2,1)

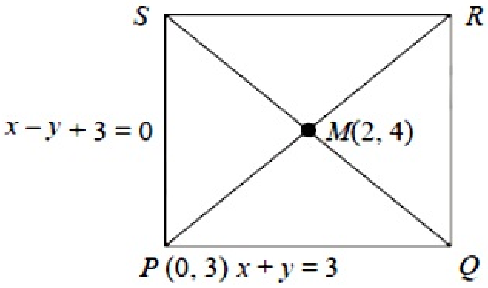
C. (2,6)

D. (3,6)

Answer: D

Solution:

Solution:



Since, $x - y + 3 = 0$ and $x + y = 3$ are perpendicular lines and intersection point of $x - y + 3 = 0$ and $x + y = 3$ is $P(0, 3)$.
$\Rightarrow M$ is mid-point of $PR \Rightarrow R(4, 5)$
Let $S(x_1, x_1 + 3)$ and $Q(x_2, 3 - x_2)$
M is mid-point of SQ
$\Rightarrow x_1 + x_2 = 4, x_1 + 3 + 3 - x_2 = 8$
$\Rightarrow x_1 = 3, x_2 = 1$
Then, the vertex D is (3,6)

Question93

If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1,2),(3,4) and (2, 5), then the equation of the diagonal AD is:
[Jan. 11, 2019 (II)]

Options:

A. $5x - 3y + 1 = 0$

B. $5x + 3y - 11 = 0$

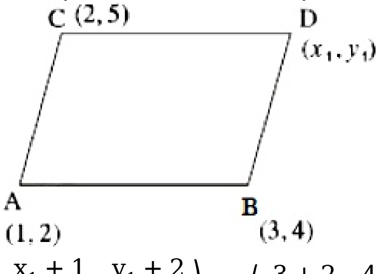
C. $3x - 5y + 7 = 0$

D. $3x + 5y - 13 = 0$

Answer: A

Solution:

Solution:
Since, in parallelogram mid points of both diagonals considers.
$\therefore$ mid-point of AD = mid-point of BC



$$\left(\frac{x_1 + 1}{2}, \frac{y_1 + 2}{2}\right) = \left(\frac{3 + 2}{2}, \frac{4 + 5}{2}\right)$$
$\therefore (x_1, y_1) = (4, 7)$
Then, equation of AD is,
$$y - 7 = \frac{2 - 7}{1 - 4}(x - 4)$$
$$y - 7 = \frac{5}{3}(x - 4)$$
$3y - 21 = 5x - 20$
$5x - 3y + 1 = 0$

Question94

If a straight line passing through the point P(−3, 4) is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is:
[Jan. 12, 2019 (II)]

Options:

A. $3x - 4y + 25 = 0$

B. $4x - 3y + 24 = 0$

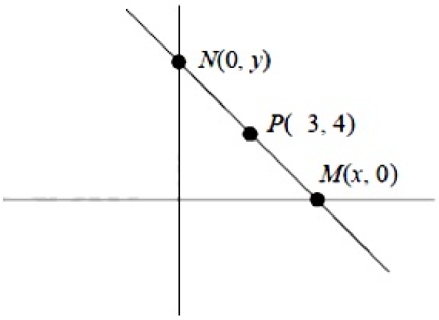
C. $x - y + 7 = 0$

D. $4x + 3y = 0$

Answer: B

Solution:

Solution:
Since, P is mid point of MN



Then, $\frac{0 + x}{2} = -3$
$\Rightarrow x = -3 \times 2 \Rightarrow x = -6$
and $\frac{y + 0}{2} = 4 \Rightarrow y + 0 = 2 \times 4 \Rightarrow y = 8$
Hence required equation of straight line MN is
$\frac{x}{-6} + \frac{y}{8} = 1 \Rightarrow 4x - 3y + 24 = 0$

Question95

If the straight line, 2x − 3y + 17 = 0 is perpendicular to the line passing through the points (7,17) and (15, β), then β equals:
[Jan. 12, 2019 (I)]

Options:

A. $\frac{35}{3}$

B. -5

C. $-\frac{35}{3}$

D. 5

Answer: D

Solution:

Solution:
∵ Equation of straight line can be rewritten as,
$y = \frac{2}{3}x + \frac{17}{3}$
∴ Slope of straight line $= \frac{2}{3}$
Slope of line passing through the points (7,17) and (15, β)
$= \frac{\beta - 17}{15 - 7} = \frac{\beta - 17}{8}$
Since, lines are perpendicular to each other.
Hence, $m_1 m_2 = -1$
$\Rightarrow \left(\frac{2}{3}\right)\left(\frac{\beta - 17}{8}\right) = -1 \Rightarrow \beta = 5$

Question96

Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of △AOP is 4, is : [April 8, 2019 (I)]

Options:

A. $8x^2 - 9y^2 + 9y = 18$

B. $9x^2 - 8y^2 + 8y = 16$

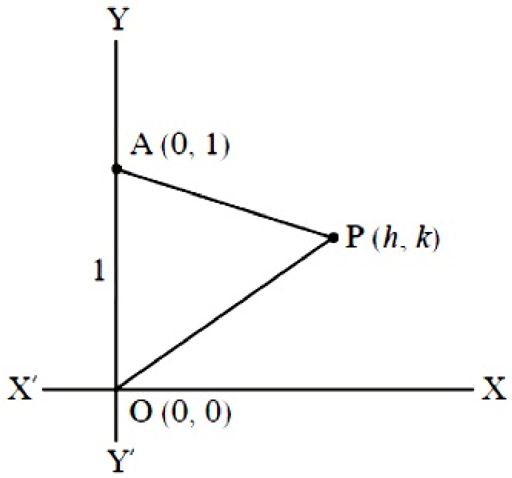
C. $9x^2 + 8y^2 - 8y = 16$

D. $8x^2 + 9y^2 - 9y = 18$

Answer: C

Solution:

Solution:
Let point P(h, k)
∵ $OA = 1$
So, $OP + AP = 3$
$\Rightarrow \sqrt{h^2 + k^2} + \sqrt{h^2 + (k-1)^2} = 3$



$\Rightarrow h^2 + (k-1)^2 = 9 + h^2 + k^2 - 6\sqrt{h^2 + k^2}$
$\Rightarrow 6\sqrt{h^2 + k^2} = 2k + 8$
$\Rightarrow 9h^2 + 8k^2 - 8k - 16 = 0$
Hence, locus of point P is
$9x^2 + 8y^2 - 8y - 16 = 0$

Question97

Slope of a line passing through P(2, 3) and intersecting the line $x + y = 7$ at a distance of 4 units from P, is: [April 9, 2019 (I)]

Options:

A. $\frac{1-\sqrt{5}}{1+\sqrt{5}}$

B. $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

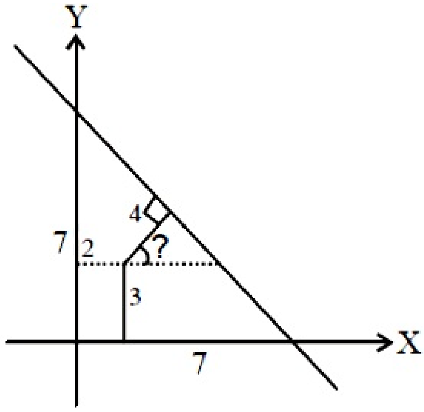
C. $\frac{\sqrt{7}-1}{\sqrt{7}+1}$

D. $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

Answer: B

Solution:

Solution:



Since point at 4 units from P(2, 3) will be
A $(4\cos\theta + 2, 4\sin\theta + 3)$ and this point will satisfy the equation of line $x + y = 7$

$\Rightarrow \cos\theta + \sin\theta = \frac{1}{2}$

On squaring

$\Rightarrow \sin 2\theta - \frac{3}{4} \Rightarrow \frac{2\tan\theta}{1+\tan^2\theta} = -\frac{3}{4}$

$\Rightarrow 3\tan^2\theta + 8\tan\theta + 3 = 0$

$\Rightarrow \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6}$ (ignoring -ve sign)

$\Rightarrow \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6} = \frac{1-\sqrt{7}}{1+\sqrt{7}}$

Question98

A point on the straight line, 3x + 5y = 15 which is equidistant from the coordinate axes will lie only in :
[April 8, 2019 (I)]

Options:

A. 4^{th} quadrant

B. 1^{st} quadrant

C. 1^{st} and 2^{nd} quadrants

D. 1^{st}, 2^{nd} and 4^{th} quadrants

Answer: C

Solution:

Solution:

A point which is equidistant from both the axes lies on either $y = x$ and $y = -x$.

Since, point lies on the line $3x + 5y = 15$

Then the required point

$3x + 5y = 15$

$x + y = 0$

$x = -\frac{15}{2}$

$y = \frac{15}{2} \Rightarrow (x, y) = \left(-\frac{15}{2}, \frac{15}{2}\right)\{2^{nd}$ quadrant $\}$

$3x + 5y = 15$

or $\frac{x - y = 0}{15}$

$x = \frac{15}{8}$

$y = \frac{15}{8} \Rightarrow (x, y) = \left(\frac{15}{8}, \frac{15}{8}\right)\{1^{st}$ quadrant $\}$

Hence, the required point lies in 1^{st} and 2^{nd} quadrant.

Question99

Two vertical poles of heights, 20m and 80m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is:

[April 08, 2019 (II)]

Options:

A. 15

B. 18

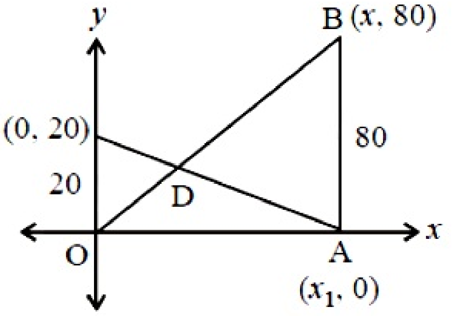
C. 12

D. 16

Answer: D

Solution:

Solution:



Equations of lines OB and AC are respectively

$y = \frac{80}{x_1}x$ . . . (i)

$\frac{x}{x_1} + \frac{y}{20} = 1$ . . . (ii)

∵ equations (i) and (ii) intersect each other

∴ substitute the value of x from equation (i) to equation (ii), we get

$\frac{y}{80} + \frac{y}{20} = 1$

$\Rightarrow y + 4y = 80 \Rightarrow y = 16m$

Hence, height of intersection point is $16m$.

Question100

Suppose that the points (h, k), (1, 2) and (-3,4) lie on the line L_1. If a line L_2 passing through the points (h, k) and (4,3) is perpendicular on L_1, then $\frac{k}{h}$ equals:

[April 08, 2019 (II)]

Options:

A. $\frac{1}{3}$

B. 0

C. 3

D. $-\frac{1}{7}$

Answer: A

Solution:

Solution:
$\because$(h, k), (1, 2) and (-3,4) are collinear

$$\therefore \begin{vmatrix} h & k & 1 \\ 1 & 2 & 1 \\ -3 & 4 & 1 \end{vmatrix} = 0 \Rightarrow -2h - 4k + 10 = 0$$

$\Rightarrow h + 2k = 5 \ldots$ (i)

Now, $m_{L_1} = \frac{4-2}{-3-1} = -\frac{1}{2} \Rightarrow m_{L_2} = 2 \ [\because L_1 \perp L_2]$

By the given points (h, k) and (4,3) ,

$m_{L_2} = \frac{k-3}{h-4} \Rightarrow \frac{k-3}{h-4} = 2 \Rightarrow k - 3 = 2h - 8$

$2h - k = 5 \ldots$ (ii)

From (i) and (ii)

$h = 3, k = 1 \Rightarrow \frac{k}{h} = \frac{1}{3}$

Question101

If the two lines $x + (a-1)y = 1$ and $2x + a^2 y = 1 (a \in R - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is: [April 09, 2019 (II)]

Options:

A. $\sqrt{\frac{2}{5}}$

B. $\frac{2}{5}$

C. $\frac{2}{\sqrt{5}}$

D. $\frac{\sqrt{2}}{5}$

Answer: A

Solution:

Solution:
$\because$ two lines are perpendicular $\Rightarrow m_1 m_2 = -1$

$\Rightarrow \left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$

$\Rightarrow 2 = a^2(1-a) \Rightarrow a^3 - a^2 + 2 = 0$

$\Rightarrow (a+1)(a^2 + 2a + 2) = 0 \Rightarrow a = -1$

Hence equations of lines are $x - 2y = 1$ and $2x + y = 1$

$\therefore$ intersection point is $\left(\frac{3}{5}, \frac{-1}{5}\right)$

Now, distance from origin $= \sqrt{\frac{9}{25} + \frac{1}{25}} = \sqrt{\frac{10}{25}} = \sqrt{\frac{2}{5}}$

Question102

A rectangle is inscribed in a circle with a diameter lying along the line 3y = x + 7. If the two adjacent vertices of the rectangle are (-8,5) and (6, 5), then the area of the rectangle (in sq. units) is:
[April 09, 2019 (II)]

Options:

A. 84

B. 98

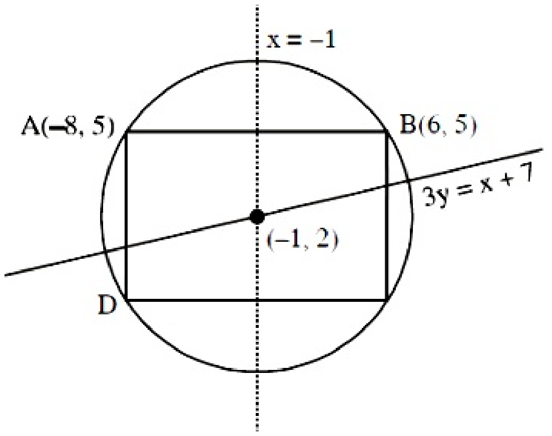
C. 72

D. 56

Answer: A

Solution:

Solution:
Given situation



∴ perpendicular bisector of AB will pass from centre.
∴ equation of perpendicular bisector $x = -1$
Hence centre of the circle is (-1,2)
Let co-ordinate of D is (α, β)
$\Rightarrow \frac{\alpha + 6}{2} = -1$ and $\frac{\beta + 5}{2} = 2$
$\Rightarrow \alpha = -8$ and $\beta = -1$, $\therefore D \equiv (-8, -1)$
$|AD| = 6$ and $|AB| = 14$
Area $= 6 \times 14 = 84$

Question103

Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines?
[April 10, 2019 (II)]

Options:

A. $\left(-\frac{1}{4}, \frac{2}{3}\right)$

B. $\left(\frac{1}{4}, -\frac{1}{3}\right)$

C. $\left(\frac{1}{4}, \frac{1}{3}\right)$

D. $\left(-\frac{1}{4}, -\frac{2}{3}\right)$

Answer: A

Solution:

Solution:
Let straight line be $4x - 3y + \alpha = 0$
$\because$ distance from origin $= \frac{3}{5}$
$\therefore \frac{3}{5} = \left|\frac{\alpha}{5}\right| \Rightarrow \alpha = \pm 3$
Hence, line is $4x - 3y + 3 = 0$ or $4x - 3y - 3 = 0$
Clearly $\left(-\frac{1}{4}, \frac{2}{3}\right)$ satisfies $4x - 3y + 3 = 0$

Question104

A triangle has a vertex at (1,2) and the mid points of the two sides through it are (-1,1) and (2,3) . Then the centroid of this triangle is: [April 12, 2019 (II)]

Options:

A. $\left(1, \frac{7}{3}\right)$

B. $\left(\frac{1}{3}, 2\right)$

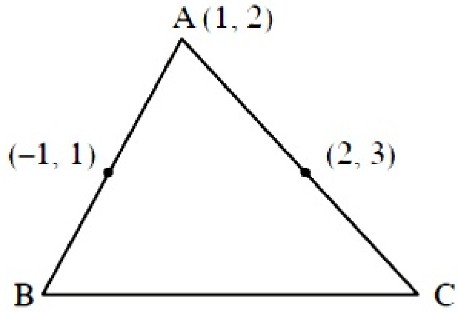
C. $\left(\frac{1}{3}, 1\right)$

D. $\left(\frac{1}{3}, \frac{5}{3}\right)$

Answer: B

Solution:

Solution:
From the mid-point formula co-ordinates of vertex B and C are $B(-3, 0)$ and $C(3, 4)$
Now, centroid of the triangle
$G \equiv \left(\frac{3 - 3 + 1}{3}, \frac{0 + 4 + 2}{3}\right) \Rightarrow G \equiv \left(\frac{1}{3}, 2\right)$



Question105

A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line $x + y = 0$. Then an equation of the line Lis:
[April 12, 2019 (II)]

Options:

A. $x + \sqrt{3}y = 8$

B. $(\sqrt{3} + 1)x + (\sqrt{3} - 1)y = 8\sqrt{2}$

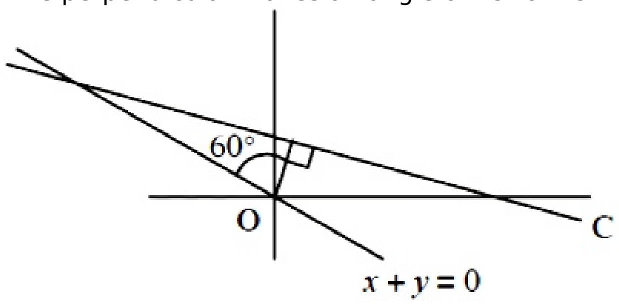
C. $\sqrt{3}x + y = 8$

D. None of these

Answer: B

Solution:

Solution:
∵ perpendicular makes an angle of 60° with the line $x + y = 0$
∴ the perpendicular makes an angle of 15° or 75° with x -axis



Hence, the equation of line will be
$x \cos 75^\circ + y \sin 75^\circ = 4$
or $x \cos 15^\circ + y \sin 15^\circ = 4$
$(\sqrt{3} - 1)x + (\sqrt{3} + 1)y = 8\sqrt{2}$
or $(\sqrt{3} + 1)x + (\sqrt{3} - 1)y = 8\sqrt{2}$

Question106

The equation $y = \sin x \sin(x + 2) - \sin^2(x + 1)$ represents a straight line lying in :

[April 12, 2019 (I)]

Options:

A. second and third quadrants only

B. first, second and fourth quadrant

C. first, third and fourth quadrants

D. third and fourth quadrants only

Answer: D

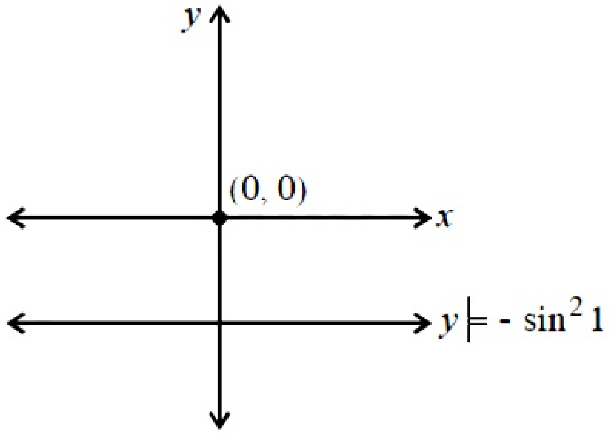
Solution:

Solution:

Consider the equation, $y = \sin x \cdot \sin(x+2) - \sin^2(x+1)$

$= \frac{1}{2}\cos(-2) - \frac{\cos(2x+2)}{2} - \left[\frac{1-\cos(2x+2)}{2}\right]$

$= \frac{(\cos 2) - 1}{2} = -\sin^2 1$



By the graph y lies in III and IV quadrant.

Question107

Let the orthocentre and centroid of a triangle be A(−3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is :
[2018]

Options:

A. $2\sqrt{10}$

B. $3\sqrt{\frac{5}{2}}$

C. $\frac{3\sqrt{5}}{2}$

D. $\sqrt{10}$

Answer: B

Solution:

Solution:
Since Orthocentre of the triangle is $A(-3, 5)$ and centriod of the triangle is $B(3, 3)$, then
$AB = \sqrt{40} = 2\sqrt{10}$

Centroid divides orthocentre and circumcentre of the triangle in ratio 2: 1
$\therefore AB : BC = 2 : 1$
$Now, AB = \frac{2}{3}AC$
$\Rightarrow AC = \frac{3}{2}AB = \frac{3}{2}(2\sqrt{10}) \Rightarrow AC = 3\sqrt{10}$
∴ Radius of circle with AC as diametre
$= \frac{AC}{2} = \frac{3}{2}\sqrt{10} = 3\sqrt{\frac{5}{2}}$

Question108

A straight line through a fixed point (2,3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is : [2018]

Options:

A. $2x + 3y = xy$

B. $3x + 2y = xy$

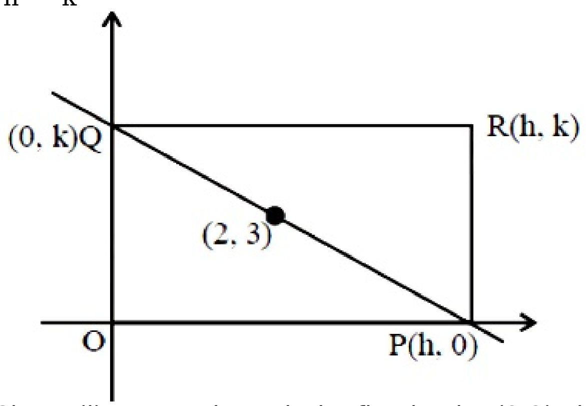
C. $3x + 2y = 6xy$

D. $3x + 2y = 6$

Answer: B

Solution:

Solution:
Equation of PQ is
$\frac{x}{h} + \frac{y}{k} = 1 \ldots$ (i)



Since, (i) passes through the fixed point (2,3) Then,
$\frac{2}{h} + \frac{3}{k} = 1$

Then, the locus of R is $\frac{2}{x} + \frac{3}{y} = 1$ or $3x + 2y = xy$.

Question109

In a triangle ABC, coordianates of A are (1,2) and the equations of the medians through B and C are $x + y = 5$ and $x = 4$ respectively. Then area of ΔABC (in sq. units) is [Online April 15, 2018]

Options:

A. 5

B. 9

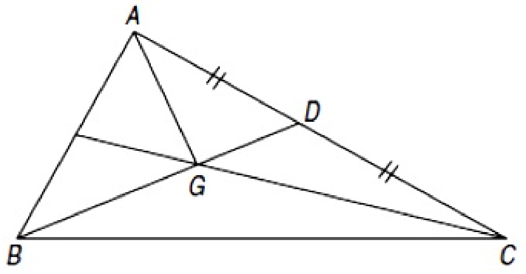
C. 12

D. 4

Answer: B

Solution:

Solution:
Median through C is $x = 4$
So the x coordinate of C is 4 . let $C \equiv (4, y)$, then the midpoint of $A(1, 2)$ and $C(4, y)$ is D which lies on the median through B.



$\therefore\ D \equiv \left(\frac{1+4}{2}, \frac{2+y}{2}\right)$

Now , $\frac{1+4+2+y}{2} = 5 \Rightarrow y = 3$

So, $C \equiv (4, 3)$

The centroid of the triangle is the intersection of the mesians. Here the medians $x = 4$ and $x + 4$ and $x + y = 5$ intersect at $G(4, 1)$

The area of triangle $\Delta ABC = 3 \times \Delta\, AGC$

$= 3 \times \frac{1}{2}[1(1-3) + 4(3-2) + 4(2-1)] = 9$

Question110

The foot of the perpendicular drawn from the origin, on the line, $3x + y = \lambda (\lambda \neq 0)$ is P. If the line meets x -axis at A and y -axis at B, then the ratio BP : PA is [Online April 15, 2018]

Options:

A. 9: 1

B. 1: 3

C. 1: 9

D. 3: 1

Answer: A

Solution:

Solution:

Let (x, y) be foot of perpendicular drawn to the point (x_1, y_1) on the line $ax + by + c = 0$

Relation : $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{-(ax_1 + by_1 + cz_1)}{a^2 + b^2}$

Here $(x_1, y_1) = (o, 0)$

given line is: $3x + y - \lambda = 0$

$\frac{x - 0}{3} = \frac{y - 0}{1} = \frac{-((3 \times 0) + (1 \times 0) - \lambda)}{3^2 + 1^2}$

$x = \frac{3\lambda}{10}$ and $y = \frac{\lambda}{10}$

Hence foot of perpendicular $P = \left(\frac{3\lambda}{10}, \frac{\lambda}{10}\right)$

Line meets X -axis at $A = \left(\frac{\lambda}{3}, 0\right)$

and meets Y -axis at $B = (0, \lambda)$

$BP = \sqrt{\left(\frac{3\lambda}{10}\right)^2 + \left(\frac{\lambda}{10} - \lambda\right)^2}$

$\Rightarrow BP = \sqrt{\frac{9\lambda^2}{100} + \frac{81\lambda^2}{100}}$

$\therefore BP = \sqrt{\frac{90\lambda^2}{100}}$

$AP = \sqrt{\left(\frac{\lambda}{3} - \frac{3\lambda}{10}\right)^2 + \left(0 - \frac{\lambda}{10}\right)^2}$

$\Rightarrow AP = \sqrt{\frac{\lambda^2}{900} + \frac{\lambda^2}{100}}$

$\therefore AP = \sqrt{\frac{10\lambda^2}{900}}$

$\therefore AP : AP = 9 : 1$

Question111

The sides of a rhombus ABCD are parallel to the lines, $x - y + 2 = 0$ and $7x - y + 3 = 0$. If the diagonals of the rhombus intersect at $P(1, 2)$ and the vertex A (different from the origin) is on the y -axis, then the ordinate of A is [Online April 15, 2018]

Options:

A. 2

B. $\frac{7}{4}$

C. $\frac{7}{2}$

D. $\frac{5}{2}$

Answer: D

Solution:

Solution:
Let the coordinate A be $(0, c)$
Equations of the given lines are
$x - y + 2 = 0$ and
$7x - y + 3 = 0$
We know that the diagonals of the rhombus will be parallel to the angle bisectors of the two given lines;
$y = x + 2$ and $y = 7x + 3$
∴ equation of angle bisectors is given as:
$\frac{x - y + 2}{\sqrt{2}} = \pm \frac{7x - y + 3}{5\sqrt{2}}$
$5x - 5y + 10 = \pm(7x - y + 3)$
∴ Parallel equations of the diagonals are $2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$
∴ slopes of diagonals are $\frac{-1}{2}$ and 2 .
Now, slope of the diagonal from $A(0, c)$ and passing through $P(1, 2)$ is $(2 - c)$
$\therefore 2 - c = 2 \Rightarrow c = 0$ (not possible)
$\therefore 2 - c = \frac{-1}{2} \Rightarrow c = \frac{5}{2}$
∴ ordinate of A is $\frac{5}{2}$

Question112

A square, of each side 2 , lies above the x -axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x -axis, then the sum of the x - coordinates of the vertices of the square is : [Online April 9, 2017]

Options:

A. $2\sqrt{3} - 1$

B. $2\sqrt{3} - 2$

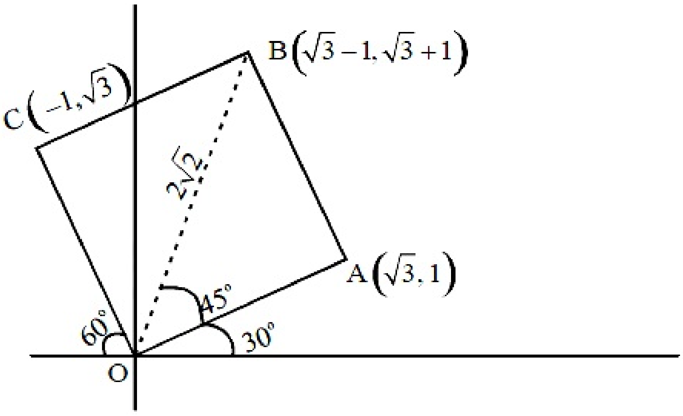
C. $\sqrt{3} - 2$

D. $\sqrt{3} - 1$

Answer: B

Solution:

Solution:



For A;

$$\frac{x}{\cos 30^\circ} = \frac{y}{\sin 30^\circ} = 2$$

$\Rightarrow x = \sqrt{3}$ and $y = 1$

For C

$$\frac{x}{\cos 120^\circ} = \frac{y}{\sin 120^\circ} = 2$$

$\Rightarrow x = -1, y = \sqrt{3}$

For B

$$\frac{x}{\cos 75^\circ} = \frac{y}{\sin 75^\circ} = 2\sqrt{2}$$

$\Rightarrow x = \sqrt{3} - 1$ and $y = \sqrt{3} + 1$

$\therefore \text{Sum} = 2\sqrt{3} - 2$

Question113

A ray of light is incident along a line which meets another line, $7x - y + 1 = 0$, at the point (0,1) . The ray is then reflected from this point along the line, $y + 2x = 1$. Then the equation of the line of incidence of the ray of light is : [Online April 10, 2016]

Options:

A. $41x - 25y + 25 = 0$

B. $41x + 25y - 25 = 0$

C. $41x - 38y + 38 = 0$

D. $41x + 38y - 38 = 0$

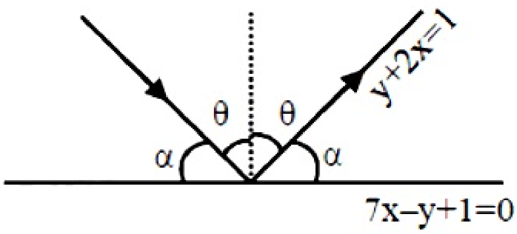
Answer: C

Solution:

Solution:

Let slope of incident ray be m

$\therefore$ angle of incidence = angle of reflection



$\therefore \left|\frac{m-7}{1+7m}\right| = \left|\frac{-2-7}{1-14}\right| = \frac{9}{13}$

$\Rightarrow \frac{m-7}{1+7m} = \frac{9}{13}$ or $\frac{m-7}{1+7m} = -\frac{9}{13}$

$\Rightarrow 13m - 91 = 9 + 63m$ or $13m - 91 = -9 - 63m$

$\Rightarrow 50m = -100$ or $76m = 82$

$\Rightarrow m = -\frac{1}{2}$ or $m = \frac{41}{38}$

$\Rightarrow y - 1 = -\frac{1}{2}(x-0)$ or $y - 1 = \frac{41}{38}(x-0)$

i.e $x + 2y - 2 = 0$ or $38y - 38 - 41x = 0$

$\Rightarrow 41x - 38y + 38 = 0$

Question114

Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus? [2016]

Options:

A. $\left(\frac{1}{3}, -\frac{8}{3}\right)$

B. $\left(-\frac{10}{3}, -\frac{7}{3}\right)$

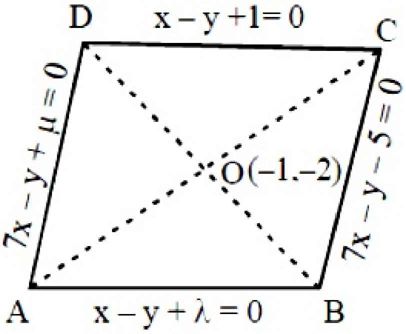
C. (-3,-9)

D. (-3,-8)

Answer: A

Solution:

Solution:



Let other two sides of rhombus are

$x - y + \lambda = 0$

and $7x - y + \mu = 0$

then O is equidistant from AB and DC and from AD and BC

$\therefore |-1 + 2 + 1| = |-1 + 2 + \lambda| \Rightarrow \lambda = -3$ and $|-7 + 2 - 5| = |-7 + 2 + \mu| \Rightarrow \mu = 15$

$\therefore$ Other two sides are $x - y - 3 = 0$ and $7x - y + 15 = 0$

$\therefore$ On solving the eq ^{n} s of sides pairwise, we get the vertices as

,(1, 2), $\left(\frac{-7}{3}, \frac{-4}{3}\right)$, $(-3, -6)$ $\left(\frac{1}{3}, \frac{-8}{3}\right)$

Question115

If a variable line drawn through the intersection of the lines $\frac{x}{3} + \frac{y}{4} = 1$ and $\frac{x}{4} + \frac{y}{3} = 1$, meets the coordinate axes at A and B, (A ≠ B), then the locus of the midpoint of AB is :
[Online April 9, 2016]

Options:

A. $7xy = 6(x + y)$

B. $4(x + y)^2 - 28(x + y) + 49 = 0$

C. $6xy = 7(x + y)$

D. $14(x + y)^2 - 97(x + y) + 168 = 0$

Answer: A

Solution:

Solution:
$L_1 : 4x + 3y - 12 = 0$
$L_2 : 3x + 4y - 12 = 0$
$L_1 + \lambda L_2 = 0$
$(4x + 3y - 12) + \lambda(3x + 4y - 12) = 0$
$x(4 + 3\lambda) + y(3 + 4\lambda) - 12(1 + \lambda) = 0$
Point A $\left(\frac{12(1 + \lambda)}{4 + 3\lambda}, 0 \right)$
Point B $\left(0, \frac{12(1 + \lambda)}{3 + 4\lambda} \right)$
mid point $\Rightarrow h = \frac{6(1 + \lambda)}{4 + 3\lambda}$ .... (i)
$k = \frac{6(1 + \lambda)}{3 + 4\lambda}$ .... (ii)
Eliminate λ from (i) and (ii), then
$6(h + k) = 7hk$
$6(x + y) = 7xy$

Question116

The point (2,1) is translated parallel to the line L : x − y = 4 by $2\sqrt{3}$ units. If the new points Q lies in the third quadrant, then the equation of the line passing through Q and perpendicular to L is :
[Online April 9, 2016]

Options:

A. $x + y = 2 - \sqrt{6}$

B. $2x + 2y = 1 - \sqrt{6}$

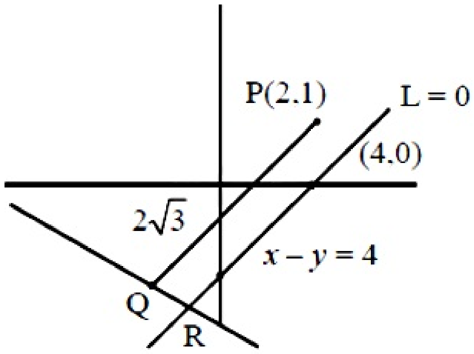
C. $x + y = 3 - 3\sqrt{6}$

D. $x + y = 3 - 2\sqrt{6}$

Answer: D

Solution:

Solution:
$x - y = 4$
To find equation of R
slope of $L = 0$ is 1
$\Rightarrow$ slope of $QR = -1$
Let QR is $y = mx + c$
$y = -x + c$
$x + y - c = 0$
distance of QR from (2,1) is $2\sqrt{3}$
$2\sqrt{3} = \frac{|2 + 1 - c|}{\sqrt{2}}$



$2\sqrt{6} = |3 - c|$
$c - 3 = \pm 2\sqrt{6} c = 3 \pm 2\sqrt{6}$
Line can be $x + y = 3 \pm 2\sqrt{6}$
$x + y = 3 - 2\sqrt{6}$

Question117

A straight line through origin O meets the lines 3y = 10 − 4x and 8x + 6y + 5 = 0 at points A and B respectively. Then O divides the segment AB in the ratio : [Online April 10, 2016]

Options:

A. 2: 3

B. 1: 2

C. 4: 1

D. 3: 4

Answer: C

Solution:

Solution:
Length of $\perp$ to $4x + 3y = 10$ from origin (0,0)
$P_1 = \frac{10}{5} = 2$
Length of $\perp$ to $8x + 6y + 5 = 0$ from origin (0,0)
$P_2 = \frac{5}{10} = \frac{1}{2}$
$\because$ Lines are parallel to each other $\Rightarrow$ ratio will be 4: 1 or 1: 4

Question118

Let L be the line passing through the point P(1, 2) such that its intercepted segment between the co-ordinate axes is bisected at P. If L_1 is the line perpendicular to L and passing through the point (−2, 1), then the point of intersection of L and L_1 is :
[Online April 10, 2015]

Options:

A. $\left(\frac{4}{5}, \frac{12}{5}\right)$

B. $\left(\frac{3}{5}, \frac{23}{10}\right)$

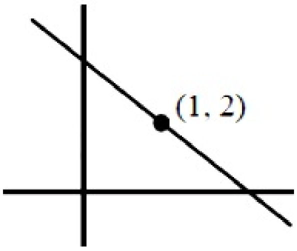
C. $\left(\frac{11}{20}, \frac{29}{10}\right)$

D. $\left(\frac{3}{10}, \frac{17}{5}\right)$

Answer: A

Solution:

Solution:
Equation of line L
$\frac{x}{2} + \frac{y}{4} = 1$
$2x + y = 4$ ... (i)



For line
$x - 2y = -4$ ...(ii)
solving equation (i) and (ii); we get point of intersection
$\left(4/5, \frac{12}{5}\right)$

Question119

The points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82,30):
[Online April 10, 2015]

Options:

A. form an acute angled triangle.

B. form a right angled triangle.

C. lie on a straight line.

D. form an obtuse angled triangle.

Answer: C

Solution:

Solution:

$A\left(0, \frac{8}{3}\right)B(1, 3)C(89, 30)$

Slope of AB = $\frac{1}{3}$

Slope of BC = $\frac{1}{3}$

So, lies on same line

Question120

A straight line L through the point (3,-2) is inclined at an angle of 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x -axis, then the equation of L is :
[Online April 11, 2015]

Options:

A. $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$

B. $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

C. $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

D. $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$

Answer: C

Solution:

Solution:

Given eqn of line is $y + \sqrt{3}x - 1 = 0$

$\Rightarrow y = -\sqrt{3}x + 1$

$\Rightarrow$(slope)$m_2 = -\sqrt{3}$

Let the other slope be m_1

$\therefore \tan 60^{\circ} = | \frac{m_1 - (-\sqrt{3})}{1 + (-\sqrt{3}m_1)}$

$\Rightarrow m_1 = 0, m_2 = \sqrt{3}$

Since line L is passing through (3,-2)

$\therefore y - (-2) = +\sqrt{3}(x - 3)$

$\Rightarrow y + 2 = \sqrt{3}(x - 3)$

$y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

Question121

The circum centre of a triangle lies at the origin and its centroid is the mid point of the line segment joining the points $(a^2 + 1, a^2 + 1)$ and $(2a, -2a)$, $a \neq 0$. Then for anya, the orthocentre of this triangle lies on

the line:
[Online April 11, 2015]

Options:

A. $y - 2ax = 0$

B. $y - (a^2 + 1)x = 0$

C. $y + x = 0$

D. $(a-1)^2 x - (a+1)^2 y = 0$

Answer: D

Solution:

Solution:
Circumcentre $= (0, 0)$
Centroid $= \left(\frac{(a+1)^2}{2}, \frac{(a-1)^2}{2} \right)$
We know the circumcentre (O),
Centroid (G) and orthocentre (H) of a triangle lie on the line joining the O and G.
Also, $\frac{HG}{GO} = \frac{2}{1}$
⇒ Coordinate of orthocentre $= \frac{3(a+1)^2}{2}, \frac{3(a-1)^2}{2}$
Now, these coordinates satisfies eqn given in option (d) Hence, required eqn of line is
$(a-1)^2 x - (a+1)^2 y = 0$

Question122

Given three points P, Q, R with P(5, 3) and R lies on the x-axis. If equation of RQ is x − 2y = 2 and PQ is parallel to the x -axis, then the centroid of ΔPQR lies on the line:
[Online April 9, 2014]

Options:

A. $2x + y - 9 = 0$

B. $x - 2y + 1 = 0$

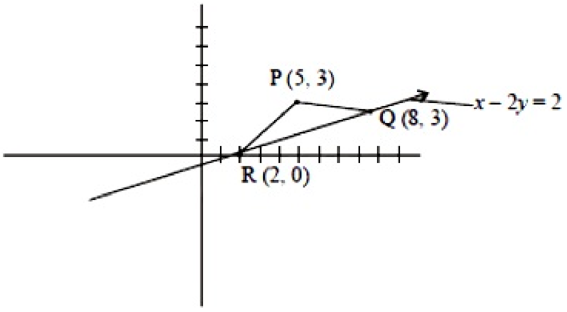
C. $5x - 2y = 0$

D. $2x - 5y = 0$

Answer: D

Solution:

Solution:



Equation of RQ is $x - 2y = 2\ldots$ (i)
at $y = 0$, $x = 2[R(2, 0)]$
as PQ is parallel to x, y -coordinates of Q is also 3 Putting value of y in equation (i), we get $Q(8, 3)$

Centroid of $\Delta PQR = \left(\frac{8+5+2}{3}, \frac{3+3}{3}\right) = (5, 2)$

Only $(2x - 5y = 0)$ satisfy the given co-ordinates.

Question123

If a line intercepted between the coordinate axes is trisected at a point A(4, 3), which is nearer to x -axis, then its equation is: [Online April 12, 2014]

Options:

A. $4x - 3y = 7$

B. $3x + 2y = 18$

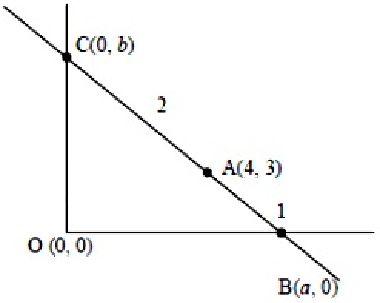
C. $3x + 8y = 36$

D. $x + 3y = 13$

Answer: B

Solution:

Solution:



A divides CB in 2: 1

$\Rightarrow 4 = \left(\frac{1 \times 0 + 2 \times a}{1+2}\right) = \frac{2a}{3}$

$\Rightarrow a = 6 \Rightarrow$ coordinate of B is $B(6, 0)$

$3 = \left(\frac{1 \times b + 2 \times 0}{1+2}\right) = \frac{b}{3}$

$\Rightarrow b = 9$ and $C(0, 9)$
Slope of line passing through (6,0),(0,9)

slope, $m = \frac{9}{-6} = -\frac{3}{2}$

Equation of line $y - 0 = \frac{-3}{2}(x - 6)$

$2y = -3x + 18$

$3x + 2y = 18$

Question124

Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then [2014]

Options:

A. $3bc - 2ad = 0$

B. $3bc + 2ad = 0$

C. $2bc - 3ad = 0$

D. $2bc + 3ad = 0$

Answer: A

Solution:

Solution:
Given lines are
$4ax + 2ay + c = 0$
$5bx + 2by + d = 0$
The point of intersection will be
$\frac{x}{2ad - 2bc} = \frac{-y}{4ad - 5bc} = \frac{1}{8ab - 10ab}$
$\Rightarrow x = \frac{2(ad - bc)}{-2ab} = \frac{bc - ad}{ab}$
$\Rightarrow y = \frac{5bc - 4ad}{-2ab} = \frac{4ad - 5bc}{2ab}$
$\because$ Point of intersection is in fourth quadrant so x is positive and y is negative.
Also distance from axes is same
So $x = -y$
($\because$ distance from x -axis is $-y$ as y is negative)
$\frac{bc - ad}{ab} = \frac{5bc - 4ad}{2ab} \Rightarrow 3bc - 2ad = 0$

Question125

Let PS be the median of the triangle vertices P(2, 2), Q(6, −1) and R(7, 3). The equation of the line passing through (1,-1) and parallel to PS is: [2014]

Options:

A. $4x + 7y + 3 = 0$

B. $2x - 9y - 11 = 0$

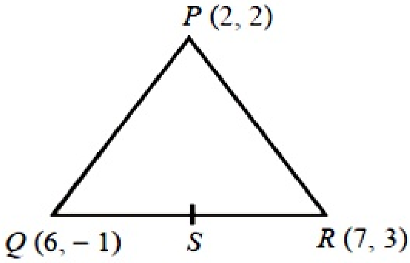
C. $4x - 7y - 11 = 0$

D. $2x + 9y + 7 = 0$

Answer: D

Solution:

Solution:
Let P, Q, R, be the vertices of ΔPQR



Since PS is the median
S is mid-point of QR

So, S = $\left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$

Now, slope of PS = $\frac{2-1}{2-\frac{13}{2}} = -\frac{2}{9}$

Since, required line is parallel to PS therefore slope of required line = slope of PS

Now, eqn. of line passing through (1,-1) and having slope $-\frac{2}{9}$ is

$y-(-1) = -\frac{2}{9}(x-1)$

$9y + 9 = -2x + 2 \Rightarrow 2x + 9y + 7 = 0$

Question126

If a line L is perpendicular to the line $5x - y = 1$, and the area of the triangle formed by the line L and the coordinate axes is 5, then the distance of line L from the line $x + 5y = 0$ is:
[Online April 19, 2014]

Options:

A. $\frac{7}{\sqrt{5}}$

B. $\frac{5}{\sqrt{13}}$

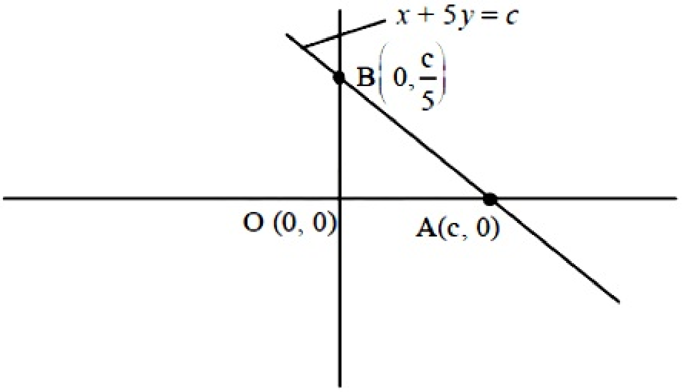
C. $\frac{7}{\sqrt{13}}$

D. $\frac{5}{\sqrt{7}}$

Answer: B

Solution:

Solution:
Let equation of line L, perpendicular to $5x - y = 1$ be $x + 5y = c$



Given that area of ΔAOB is 5 .
We know

$\{ \text{area, } A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \}$

$\Rightarrow 5 = \frac{1}{2}\left[c\left(\frac{c}{5} \right) \right]$

$\left(\begin{array}{l} \because (x_1, y_1) = (10, 0) \quad (x_3, y_3) = \left(0, \frac{c}{5}\right) \\ (x_2, y_2) = (c, 0) \end{array} \right)$

$\Rightarrow c = \pm\sqrt{50}$

Distance between L and line x + 5y = 0 is

$d = \left| \frac{\pm\sqrt{50} - 0}{\sqrt{1^2 + 5^2}} \right| = \frac{\sqrt{50}}{\sqrt{26}} = \frac{5}{\sqrt{13}}$

Question127

If the three distinct lines x + 2ay + a = 0, x + 3by + b = 0 and x + 4ay + a = 0 are concurrent, then the point (a, b) lies on a: [Online April 12, 2014]

Options:

A. circle

B. hyperbola

C. straight line

D. parabola

Answer: C

Solution:

Solution:
$x + 2ay + a = 0 \ldots$ (i)
$x + 3by + b = 0 \ldots$ (ii)
$x + 4ay + a = 0 \ldots$ (iii)
Subtracting equation (iii) from (i)
$-2ay = 0$
$ay = 0 = y = 0$
Putting value of y in equation (i), we get
$x + 0 + a = 0$
$x = -a$
Putting value of x and y in equation (ii), we get
$-a + b = 0 \Rightarrow a = b$
Thus, (a, b) lies on a straight line

Question128

The base of an equilateral triangle is along the line given by 3x + 4y = 9. If a vertex of the triangle is (1,2) , then the length of a side of the triangle is:
[Online April 11, 2014]

Options:

A. $\frac{2\sqrt{3}}{15}$

B. $\frac{4\sqrt{3}}{15}$

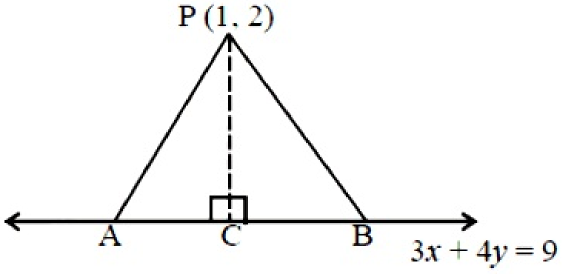
C. $\frac{4\sqrt{3}}{5}$

D. $\frac{2\sqrt{3}}{5}$

Answer: B

Solution:

Solution:



Shortest distance of a point (x_1, y_1) from line $ax + by = c$ is $d = \left|\frac{ax_1 + by_1 - c}{\sqrt{a^2 + b^2}}\right|$

Now shortest distance of $P(1, 2)$ from $3x + 4y = 9$ is $PC = d = \left|\frac{3(1) + 4(2) - 9}{\sqrt{3^2 + 4^2}}\right| = \frac{2}{5}$

Given that ΔAPB is an equilateral triangle Let ' a ' be its side then $PB = a$, $CB = \frac{a}{2}$

Now, In ΔPCB, $(PB)^2 = (PC)^2 + (CB)^2$
(By Pythagoras theoresm)

$a^2 = \left(\frac{2}{5}\right)^2 + \frac{a^2}{4}$

$a^2 - \frac{a^4}{4} = \frac{4}{25} \Rightarrow \frac{3a^2}{4} = \frac{4}{25}$

$a^2 = \frac{16}{75} \Rightarrow a = \sqrt{\frac{16}{75}} = \frac{4}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{15}$

∴ Length of Equilateral triangle (a) = $\frac{4\sqrt{3}}{15}$

Question129

The x -coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0,1)(1,1) and (1,0) is :

[2013]

Options:

A. $2+\sqrt{2}$

B. $2-\sqrt{2}$

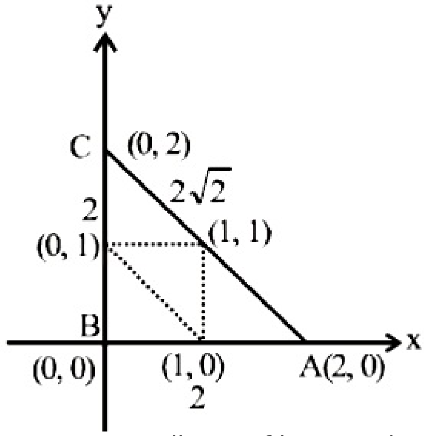
C. $1+\sqrt{2}$

D. $1-\sqrt{2}$

Answer: B

Solution:

Solution:
From the figure, we have
$a = 2, b = 2\sqrt{2}, c = 2$
$x_1 = 0, x_2 = 0, x_3 = 2$



Now, x -co-ordinate of incentre is given as
$\frac{ax_1 + bx_2 + cx_3}{a+b+c}$
⇒x -coordinate of incentre
$= \frac{2 \times 0 + 2\sqrt{2}.0 + 2.2}{2+2+2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = 2-\sqrt{2}$

Question130

A light ray emerging from the point source placed at P(1, 3) is reflected at a point Q in the axis of x. If the reflected ray passes through the point R(6, 7), then the abscissa of Q is:
[Online April 9, 2013]

Options:

A. 1

B. 3

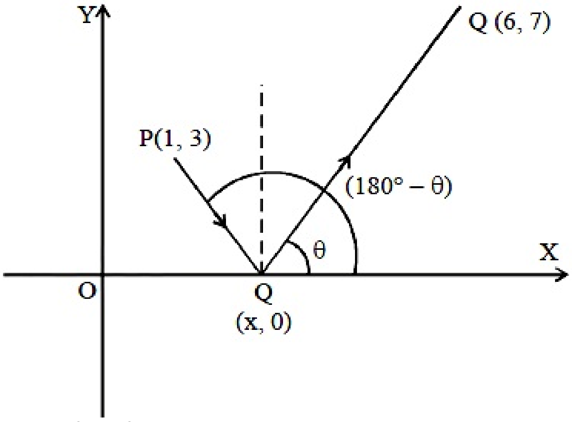
C. $\frac{7}{2}$

D. $\frac{5}{2}$

Answer: D

Solution:

Solution:
Let abcissa of Q = x



∴ Q = (x, 0)

$\tan\theta = \frac{0-7}{x-6}$, $\tan(180^\circ - \theta) = \frac{0-3}{x-1}$

Now, $\tan(180^\circ - \theta) = -\tan\theta$

$\therefore \frac{-3}{x-1} = \frac{-7}{x-6} \Rightarrow x = \frac{5}{2}$

Question131

If the three lines $x - 3y = p$, $ax + 2y = q$ and $ax + y = r$ form a right-angled triangle then : [Online April 9, 2013]

Options:

A. $a^2 - 9a + 18 = 0$

B. $a^2 - 6a - 12 = 0$

C. $a^2 - 6a - 18 = 0$

D. $a^2 - 9a + 12 = 0$

Answer: A

Solution:

Solution:
Since three lines $x - 3y = p$,
$ax + 2y = q$ and $ax + y = r$
form a right angled triangle
∴ product of slopes of any two lines $= -1$
Suppose $ax + 2y = q$ and $x - 3y = p$ are ⊥ to each other.

$\therefore \frac{-a}{2} \times \frac{1}{3} = -1 \Rightarrow a = 6$

Now, consider option one by one $a = 6$ satisfies only option (a)
∴ Required answer is $a^2 - 9a + 18 = 0$

Question132

A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x -axis, the equation of the reflected ray is
[2013]

Options:

A. $y = x + \sqrt{3}$

B. $\sqrt{3}y = x - \sqrt{3}$

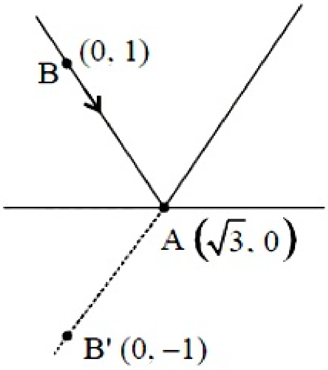
C. $y = \sqrt{3}x - \sqrt{3}$

D. $\sqrt{3}y = x - 1$

Answer: B

Solution:

Solution:
Suppose B(0, 1) be any point on given line and co-ordinate of A is $(\sqrt{3}, 0)$. So, equation of



Reflected ray is $\frac{-1-0}{0-\sqrt{3}} = \frac{y-0}{x-\sqrt{3}}$
$\Rightarrow \sqrt{3}y = x - \sqrt{3}$

Question133

If the x -intercept of some line L is double as that of the line, $3x + 4y = 12$ and the y -intercept of L is half as that of the same line, then the slope of L is :
[Online April 22, 2013]

Options:

A. -3

B. $-\frac{3}{8}$

C. $-\frac{3}{2}$

D. $-\frac{3}{16}$

Answer: D

Solution:

Solution:
Given line $3x + 4y = 12$ can be rewritten as
$\frac{3x}{12} + \frac{4y}{12} = 1 \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$
$\Rightarrow x$ -intercept $= 4$ and y -intercept $= 3$
Let the required line be
$L : \frac{x}{a} + \frac{y}{b} = 1$ where
$a = x$ -intercept and $b = y$ -intercept
According to the question
$a = 4 \times 2 = 8$ and $b = 3 / 2$
$\therefore$ Required line is $\frac{x}{8} + \frac{2y}{3} = 1$
$\Rightarrow 3x + 16y = 24$
$\Rightarrow y = \frac{-3}{16}x + \frac{24}{16}$
Hence, required slope $= \frac{-3}{16}$.

Question134

If the extremities of the base of an isosceles triangle are the points (2a, 0) and (0, a) and the equation of one of the sides is x = 2a, then the area of the triangle, in square units, is:
[Online April 23, 2013]

Options:

A. $\frac{5}{4}a^2$

B. $\frac{5}{2}a^2$

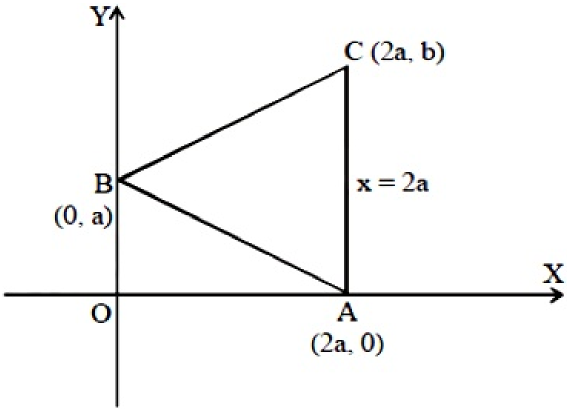
C. $\frac{25a^2}{4}$

D. $5a^2$

Answer: B

Solution:

Solution:
Let y -coordinate of $C = b$
$\therefore C = (2a, b)$



$AB = \sqrt{4a^2 + a^2} = \sqrt{5}a$

Now, $AC = BC \Rightarrow b = \sqrt{4a^2 + (b-a)^2}$

$\Rightarrow b^2 = 4a^2 + b^2 + a^2 - 2ab$

$\Rightarrow 2ab = 5a^2 \Rightarrow b = \frac{5a}{2}$

$\therefore C = \left(2a, \frac{5a}{2}\right)$

Hence area of the triangle

$$= \frac{1}{2}\begin{vmatrix} 2a & 0 & 1 \\ 0 & a & 1 \\ 2a & \frac{5a}{2} & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2a & 0 & 1 \\ 0 & a & 1 \\ 0 & \frac{5a}{2} & 0 \end{vmatrix}$$

$$= \frac{1}{2} \times 2a\left(-\frac{5a}{2}\right) = -\frac{5a^2}{2}$$

Since area is always +ve, hence area

$$= \frac{5a^2}{2} \text{sq} \cdot \text{unit}$$

Question135

Let θ_1 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_2 = 0$ and θ_2 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$, where c_1, c_2, c_3 are any real numbers:
Statement-1: If c_2 and c_3 are proportional, then $\theta_1 = \theta_2$.
Statement-2: $\theta_1 = \theta_2$ for all c_2 and c_3
[Online April 23, 2013]

Options:

A. Statement- 1 is true, Statement- 2 is true; Statement- 2 is a correct explanation of Statement- 1.

B. Statement- 1 is true, Statement- 2 is true; Statement- 2 is not a correct explanation of Statement- 1 .

C. Statement- 1 is false; Statement- 2 is true.

D. Statement- 1 is true; Statement- 2 is false.

Answer: A

Solution:

Solution:
Two lines $-x + 5y + c_2 = 0$ and $-x + 5y + c_3 = 0$ are parallel to each other. Hence statement- 1 is true, statement2 is true and statement- 2 is the correct explanation of statement-1.

Question136

Let A(−3, 2) and B(−2, 1) be the vertices of a triangle ABC. If the centroid of this triangle lies on the line 3x + 4y + 2 = 0, then the vertex C lies on the line :
[Online April 25, 2013]

Options:

A. $4x + 3y + 5 = 0$

B. $3x + 4y + 3 = 0$

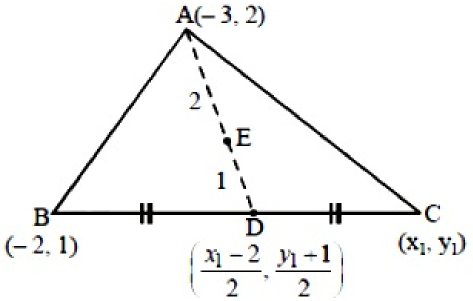
C. $4x + 3y + 3 = 0$

D. $3x + 4y + 5 = 0$

Answer: B

Solution:

Solution:
Let $C = (x_1, y_1)$



Centroid, E $= \left(\frac{x_1 - 5}{3}, \frac{y_1 + 3}{3}\right)$
Since centroid lies on the line
$3x + 4y + 2 = 0$
$\therefore\ 3\left(\frac{x_1 - 5}{3}\right) + 4\left(\frac{y_1 + 3}{3}\right) + 2 = 0$
$\Rightarrow 3x_1 + 4y_1 + 3 = 0$
Hence vertex (x_1, y_1) lies on the line
$3x + 4y + 3 = 0$

Question137

If the image of point P(2, 3) in a line L is Q(4, 5), then the image of point R(0, 0) in the same line is:
[Online April 25, 2013]

Options:

A. (2,2)

B. (4,5)

C. (3,4)

D. (7,7)

Answer: D

Solution:

Solution:
Mid-point of $P(2, 3)$ and $Q(4, 5) = (3, 4)$ Slope of $PQ = 1$
Slope of the line $L = -1$
Mid-point (3,4) lies on the line L.
Equation of line L
$y - 4 = -1(x - 3) \Rightarrow x + y - 7 = 0$
Let image of point $R(0, 0)$ be $S(x_1, y_1)$
Mid-point of $RS = \left(\frac{x_1}{2}, \frac{y_1}{2} \right)$
Mid-point $\left(\frac{x_1}{2}, \frac{y_1}{2} \right)$
lies on the line (i)
$\therefore x_1 + y_1 = 14$
Slope of $RS = \frac{y_1}{x_1}$
Since $RS \perp$ line L
$\therefore \frac{y_1}{x_1} \times (-1) = -1$
$\therefore x_1 = y_1$
From (ii) and (iii),
$x_1 = y_1 = 7$
Hence the image of $R = (7, 7)$

Question138

If the line 2x + y = k passes through the point which divides the line segment joining the points (1,1) and (2,4) in the ratio 3 : 2, then k equals :
[2012]

Options:

A. $\frac{29}{5}$

B. 5

C. 6

D. $\frac{11}{5}$

Answer: C

Solution:

Solution:
Let the points be $A(1, 1)$ and $B(2, 4)$. Let point C divides line AB in the ratio 3: 2 . So, by section formula we have

$C = \left(\frac{3 \times 2 + 2 \times 1}{3 + 2}, \frac{3 \times 4 + 2 \times 1}{3 + 2} \right) = \left(\frac{8}{5}, \frac{14}{5} \right)$

Since Line $2x + y = k$ passes through $C\left(\frac{8}{5}, \frac{14}{5} \right)$

$\Rightarrow \frac{2 \times 8}{5} + \frac{14}{5} = k \Rightarrow k = 6$

Question139

If the straight lines x + 3y = 4, 3x + y = 4 and x + y = 0 form a triangle, then the triangle is
[Online May 7, 2012]

Options:

A. scalene

B. equilateral triangle

C. isosceles

D. right angled isosceles

Answer: C

Solution:

Solution:
Let equation of $AB : x + 3y = 4$
Let equation of $BC : 3x + y = 4$
Let equation of $CA : x + y = 0$
Now, By solving these equations we get
$A = (-2, 2), B = (1, 1)$ and $C = (2, -2)$
Now, $AB = \sqrt{9 + 1} = \sqrt{10}$
$BC = \sqrt{1 + 9} = \sqrt{10}$
and $CA = \sqrt{16 + 16} = \sqrt{32}$
Since, length of AB and BC are same therefore triangle is isosceles.

Question140

If two vertical poles 20m and 80m high stand apart on a horizontal plane, then the height (in m) of the point of intersection of the lines joining the top of each pole to the foot of other is
[Online May 7, 2012]

Options:

A. 16

B. 18

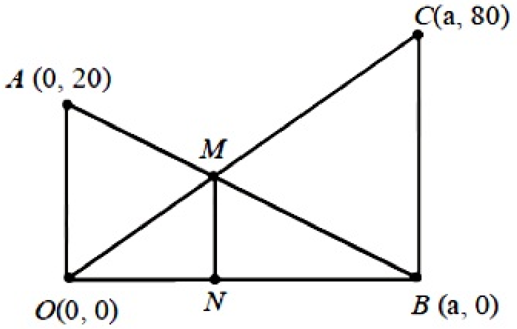
C. 50

D. 15

Answer: A

Solution:

Solution:



We put one pole at origin.
BC = 80m, OA = 20m
Line OC and AB intersect at M .
To find: Length of M N .
Eqn of OC : $y = \left(\frac{80-0}{a-0}\right)x$
$\Rightarrow\ y = \frac{80}{a}x\ \ldots$ (i)
Eqn of AB : $y = \left(\frac{20-0}{0-a}\right)(x-a)$
$\Rightarrow y = \frac{-20}{a}(x-a)\ \ldots$ (ii)
At M : (i) = (ii)
$\Rightarrow \frac{80}{a}x = \frac{-20}{a}(x-a)$
$\Rightarrow \frac{80}{a}x = \frac{-20}{a}x + 20 \Rightarrow x = \frac{a}{5}$
$\therefore\ y = \frac{80}{a} \times \frac{a}{5} = 16$

Question141

The point of intersection of the lines $(a^3+3)x + ay + a - 3 = 0$ and $(a^5+2)x + (a+2)y + 2a + 3 = 0$ (a real) lies on the y-axis for [Online May 7, 2012]

Options:

A. no value of a

B. more than two values of a

C. exactly one value of a

D. exactly two values of a

Answer: A

Solution:

Solution:
Given equation of lines are
$(a^3+3)x + ay + a - 3 = 0$ and $(a^5+2)x + (a+2)y + 2a + 3 = 0$(a real)
Since point of intersection of lines lies on y-axis.
∴ Put $x = 0$ in each equation, we get
$ay + a - 3 = 0$ and $(a+2)y + 2a + 3 = 0$
On solving these we get

$(a + 2)(a - 3) - a(2a + 3) = 0$
$\Rightarrow a^2 - a - 6 - 2a^2 - 3a = 0$
$\Rightarrow -a^2 - 4a - 6 = 0 \Rightarrow a^2 + 4a + 6 = 0$
$\Rightarrow a = \dfrac{-4 \pm \sqrt{16 - 24}}{2} = \dfrac{-4 \pm \sqrt{-8}}{2}$
(not real)
This shows that the point of intersection of the lines lies on the y -axis for no value of ' a '.

Question142

If the point (1, a) lies between the straight lines $x + y = 1$ and $2(x + y) = 3$ then a lies in interval [Online May 12, 2012]

Options:

A. $\left(\frac{3}{2}, \infty\right)$

B. $\left(1, \frac{3}{2}\right)$

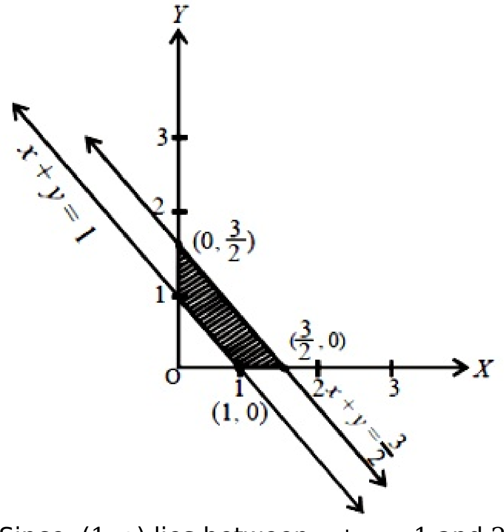
C. $(-\infty, 0)$

D. $\left(0, \frac{1}{2}\right)$

Answer: D

Solution:

Solution:



Since, (1, a) lies between $x + y = 1$ and $2(x + y) = 3$
$\therefore$ Put $x = 1$ in $2(x + y) = 3$
We get the range of y. Thus,
$2(1 + y) = 3 \Rightarrow y = \frac{3}{2} - 1 = \frac{1}{2}$
Thus 'a'lies in $\left(0, \frac{1}{2}\right)$

Question143

If two vertices of a triangle are (5,-1) and (-2,3) and its orthocentre is at (0, 0), then the third vertex is
[Online May 12, 2012]

Options:

A. (4,-7)

B. (-4,-7)

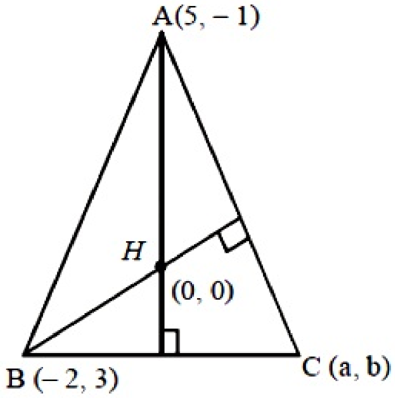
C. (-4,7)

D. (4,7)

Answer: B

Solution:

Solution:



Let the third vertex of ΔABC be (a, b).
Orthocentre $= H(0, 0)$
Let $A(5, -1)$ and $B(-2, 3)$ be other two vertices of ΔABC. Now, (Slope of AH) × (Slope of BC) $= -1$

$\Rightarrow \left(\frac{-1-0}{5-0}\right)\left(\frac{b-3}{a+2}\right) = -1$

$\Rightarrow b - 3 = 5(a + 2)$ . . . (i)

Similarly,
(Slope of BH) × (Slope of AC) $= -1$

$\Rightarrow -\left(\frac{3}{2}\right) \times \left(\frac{b+1}{a-5}\right) = -1$

$\Rightarrow 3b + 3 = 2a - 10$

$\Rightarrow 3b - 2a + 13 = 0$ . . . (ii)

On solving equations (i) and (ii) we get
$a = -4, b = -7$
Hence, third vertex is (-4,-7)

Question144

Let L be the line y = 2x, in the two dimensional plane.

Statement 1: The image of the point (0,1) in L is the point $\left(\frac{4}{5}, \frac{3}{5}\right)$

Statement 2: The points (0,1) and $\left(\frac{4}{5}, \frac{3}{5}\right)$ **lie on opposite sides of the line L and are at equal distance from it.**
[Online May 19, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1 .

C. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

D. Statement 1 is false, Statement 2 is true.

Answer: C

Solution:

Solution:
Statement - 1
Let $P'(x_1, y_1)$ be the image of (0,1) with respect to the line $2x - y = 0$ then

$$\frac{x_1}{2} = \frac{y_1 - 1}{-1} = \frac{-4(0) + 2(1)}{5}$$

$$\Rightarrow x_1 = \frac{4}{5}, y_1 = \frac{3}{5}$$

Thus, statement- 1 is true.
Also, statement- 2 is true and correct explanation for statement- 1

Question145

The line parallel to x -axis and passing through the point of intersection of lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$ where $(a, b) \neq (0, 0)$ is [Online May 26, 2012]

Options:

A. above x -axis at a distance 2 / 3 from it

B. above x -axis at a distance 3 / 2 from it

C. below x -axis at a distance 3 / 2 from it

D. below x -axis at a distance 2 / 3 from it

Answer: C

Solution:

Solution:
Given lines are
$ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$
Since, required line is || to x -axis
$\therefore x = 0$ We put $x = 0$ in given equation, we get

$$2by = -3b \Rightarrow y = -\frac{3}{2}$$

This shows that the required line is below x -axis at a distance of $\frac{3}{2}$ from it.

Question146

Consider the straight lines
$L_1 : x - y = 1$
$L_2 : x + y = 1$
$L_3 : 2x + 2y = 5$
$L_4 : 2x - 2y = 7$
The correct statement is
[Online May 26, 2012]

Options:

A. $L_1 \| L_4$, $L_2 \| L_3$, L_1 intersect L_4

B. $L_1 \perp L_2$, $L_1 \| L_3$, L_1 intersect L_2

C. $L_1 \perp L_2$, $L_2 \| L_3$, L_1 intersect L_4

D. $L_1 \perp L_2$, $L_1 \perp L_3$, L_2 intersect L_4

Answer: D

Solution:

Solution:
Consider the lines
$L_1 : x - y = 1$
$L_2 : x + y = 1$
$L_3 : 2x + 2y = 5$
$L_4 : 2x - 2y = 7$
$L_1 \perp L_2$ is correct statement
($\because$ Product of their slopes $= -1$)
$L_1 \perp L_3$ is also correct statement
($\because$ Product of their slopes $= -1$)
Now, $L_2 : x + y = 1$
$L_4 : 2x - 2y = 7$
$\Rightarrow 2x - 2(1 - x) = 7$
$\Rightarrow 2x - 2 + 2x = 7$
$\Rightarrow x = \frac{9}{4}$ and $y = \frac{-5}{4}$
Hence, L_2 intersects L_4

Question147

If a, b, c $\in$ R and 1 is a root of equation $ax^2 + bx + c = 0$, then the curve $y = 4ax^2 + 3bx + 2c$, $a \neq 0$ intersect x -axis at
[Online May 26, 2012]

Options:

A. two distinct points whose coordinates are always rational numbers

B. no point

C. exactly two distinct points

D. exactly one point

Answer: D

Solution:

Solution:
Given $ax^2 + bx + c = 0$
$\Rightarrow ax^2 = -bx - c$
Now, consider
$y = 4ax^2 + 3bx + 2c$
$= 4[-bx - c] + 3bx + 2c$
$= -4bx - 4c + 3bx + 2c = -bx - 2c$
Since, this curve intersects x -axis
$\therefore$ put $y = 0$, we get
$-bx - 2c = 0 \Rightarrow -bx = 2c$
$\Rightarrow x = \frac{-2c}{b}$
Thus, given curve intersects x -axis at exactly one point.

Question148

If A(2, −3) and B(−2, 1) are two vertices of a triangle and third vertex moves on the line 2x + 3y = 9, then the locus of the centroid of the triangle is : [2011RS]

Options:

A. $x - y = 1$

B. $2x + 3y = 1$

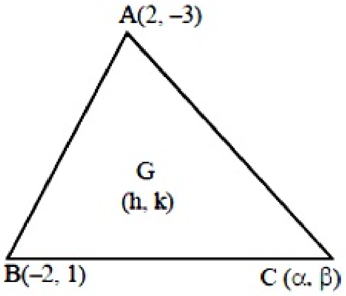
C. $2x + 3y = 3$

D. $2x - 3y = 1$

Answer: B

Solution:

Solution:



Centroid $(h, k) = \left(\frac{2 - 2 + \alpha}{3}, \frac{-3 + 1 + \beta}{3}\right)$
$\therefore \alpha = 3h$
$\beta - 2 = 3k$
$\beta = 3k + 2$
Third vertex (α, β) lies on the line
$2x + 3y = 9$
$2\alpha + 3\beta = 9$
$2(3h) + 3(3k + 2) = 9$

$2h + 3k = 1$
$2x + 3y = 1$

Question149

The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a in the interval: [2011RS]

Options:

A. $(0, \infty)$

B. $[1, \infty)$

C. $(-1, \infty)$

D. (-1,1)

Answer: B

Solution:

Solution:
Given that $x + y = |a|$ and $ax - y = 1$
Case I: If $a > 0$
$x + y = a$ . . . (i)
$ax - y = 1$ . . . (ii)
On adding equations (i) and (ii), we get
$x(1 + a) = 1 + a \Rightarrow x = 1$
y=a-1
Since given that intersection point lies in first quadrant
So, $a - 1 \geq 0$
$\Rightarrow a \geq 1$
$\Rightarrow a \in [1, \infty)$
Case II : If $a < 0$
$x + y = -a$ . . . (iii)
$ax - y = 1$ . . . (iv)
On adding equations (iii) and (iv), we get
$x(1 + a) = 1 - a$
$x = \frac{1 - a}{1 + a} > 0 \Rightarrow \frac{a - 1}{a + 1} < 0$
Since $a - 1 < 0$
$\therefore a + 1 > 0$
$\Rightarrow a > -1$ . . . (v)
$\leftarrow \frac{0}{-1} >$
$y = -a - \frac{1 - a}{1 + a} > 0 = \frac{-a - a^2 - 1 + a}{1 + a} > 0$
$\Rightarrow -\left(\frac{a^2 + 1}{a + 1}\right) > 0 \Rightarrow \frac{a^2 + 1}{a + 1} < 0$
Since $a^2 + 1 > 0$
$\therefore a + 1 < 0$
$\Rightarrow a < -1$ . . . (vi)
From (v) and (vi), $a \in \varphi$
Hence, Case-II is not possible.
So, correct answer is $a \in [1, \infty)$

Question150

The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.
Statement-1: The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$
Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.
[2011]

Options:

A. Statement- 1 is true, Statement- 2 is true; Statement- 2 is not a correct explanation for Statement- 1 .

B. Statement- 1 is true, Statement- 2 is false.

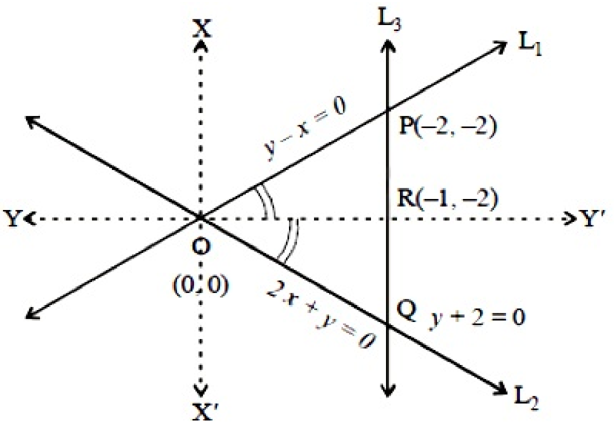
C. Statement- 1 is false, Statement- 2 is true.

D. Statement- 1 is true, Statement- 2 is true; Statement- 2 is a correct explanation for Statement-1.

Answer: B

Solution:

Solution:



$L_1 : y - x = 0$
$L_2 : 2x + y = 0$
$L_3 : y + 2 = 0$
On solving the equation of line L_1 and L_2 we get their point of intersection (0,0) i.e., origin O.
On solving the equation of line L_1 and L_3, we get $P = (-2, -2)$.
Similarly, solving equation of line L_2 and L_3, we get $Q = (-1, -2)$
We know that bisector of an angle of a triangle, divide the opposite side the triangle in the ratio of the sides including the angle [Angle Bisector Theorem of a Triangle]

$$\therefore \frac{PR}{RQ} = \frac{OP}{OQ} = \frac{\sqrt{(-2)^2 + (-2)^2}}{\sqrt{(-1)^2 + (-2)^2}} = \frac{2\sqrt{2}}{\sqrt{5}}$$

∴ Statement 1 is true but $\angle OPR \neq \angle OQR$
So ΔOPR and ΔOQR not similar
∴ Statement 2 is false

Question151

The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for :

[2009]

Options:

A. exactly one values of p

B. exactly two values of p

C. more than two values of p

D. no value of p

Answer: A

Solution:

Solution:
Given that the lines $p(p^2+1)x - y + q = 0$ and $(p^2+1)^2x + (p^2+1)y + 2q = 0$ are perpendicular to a common line then these lines must be parallel to each other,
$\therefore\ m_1 = m_2$
$\Rightarrow -\frac{p(p^2+1)}{-1} = -\frac{(p^2+1)^2}{p^2+1}$
$\Rightarrow (p^2+1)^2(p+1) = 0$
$\Rightarrow p = -1$
$\therefore$p can have exactly one value.

Question152

The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is : [2009]

Options:

A. $\frac{2\sqrt{3}}{8}$

B. $\frac{3\sqrt{2}}{5}$

C. $\frac{\sqrt{3}}{4}$

D. $\frac{3\sqrt{2}}{8}$

Answer: D

Solution:

Solution:
Let (a^2, a) be the point of shortest distance on $x = y^2$ Then distance between (a^2, a) and line $x - y + 1 = 0$ is given by
$D = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right| = \frac{|a^2 - a + 1|}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left|\left(a - \frac{1}{2}\right)^2 + \frac{3}{4}\right|$
It is min when $a = \frac{1}{2}$ and $D_{min} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

Question153

The perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y-intercept -4. Then a possible value of k is [2008]

Options:

A. 1

B. 2

C. -2

D. -4

Answer: D

Solution:

Solution:

Slope of PQ = $\frac{3-4}{k-1} = \frac{-1}{k-1}$

$\therefore$ Slope of perpendicular bisector of PQ = $(k-1)$

Also, mid point of PQ $\left(\frac{k+1}{2}, \frac{7}{2}\right)$.

$\therefore$ Equation of perpendicular bisector of PQ is

$y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right)$

$\Rightarrow 2y - 7 = 2(k-1)x - (k^2 - 1)$

$\Rightarrow 2(k-1)x - 2y + (8 - k^2) = 0$

Given that y-intercept

$= \frac{8-k^2}{2} = -4$

$\Rightarrow 8 - k^2 = -8$ or $k^2 = 16 \Rightarrow k = \pm 4$

Question154

Let A(h, k), B(1, 1) and C(2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is lsquare unit, then the set of values which 'k' can take is given by [2007]

Options:

A. $\{-1, 3\}$

B. $\{-3, -2\}$

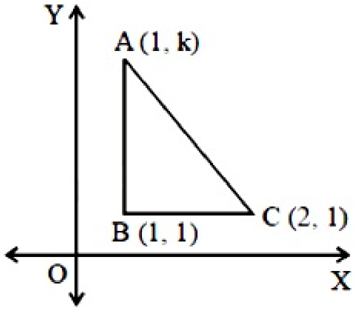
C. $\{1, 3\}$

D. $\{0, 2\}$

Answer: A

Solution:

Solution:
Given :A(1, k), B(1, 1) and C(2, 1) are vertices of a right angled triangle and area of ΔABC = 1 square unit



We know that, area of right angled triangle
$= \frac{1}{2} \times BC \times AB = 1 = \frac{1}{2}(a) \mid (k - 1)$
$\Rightarrow \pm(k - 1) = 2 \Rightarrow k = -1, 3$

Question155

Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three point. The equation of the bisector of the angle PQR is [2007]

Options:

A. $\frac{\sqrt{3}}{2}x + y = 0$

B. $x + \sqrt{3y} = 0$

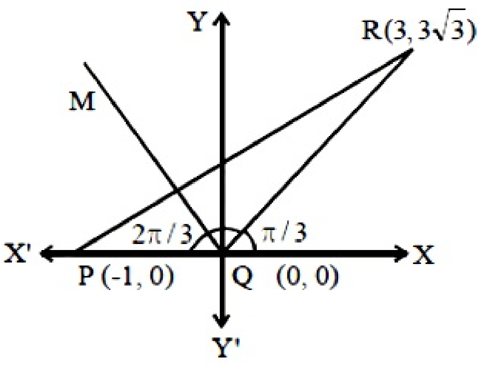
C. $\sqrt{3}x + y = 0$

D. $x + \frac{\sqrt{3}}{2}y = 0$

Answer: C

Solution:

Solution:
Given : The coordinates of points P, Q, R are $(-1, 0)$, $(0, 0)$, $(3, 3\sqrt{3})$ respectively.



Slope of QR $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3\sqrt{3}}{3}$

$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$

$\Rightarrow \angle RQX = \frac{\pi}{3}$

$\therefore \angle RQP = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Let QM bisects the $\angle PQR$,

$\therefore \angle MQR = \frac{\pi}{3} \Rightarrow \angle MQX = \frac{2\pi}{3}$

$\therefore$ Slope of the line QM $= \tan\frac{2\pi}{3} = -\sqrt{3}$

$\therefore$ Equation of line QM is $(y - 0) = -\sqrt{3}(x - 0)$

$\Rightarrow y = -\sqrt{3}x \Rightarrow \sqrt{3}x + y = 0$

Question156

If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines xy = 0, then m is [2007]

Options:

A. 1

B. 2

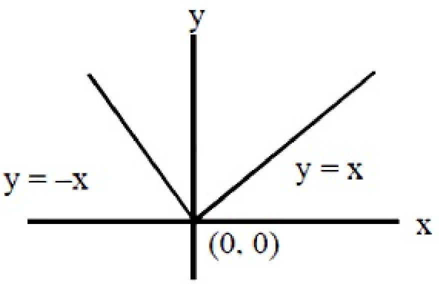
C. $-\frac{1}{2}$

D. -2

Answer: A

Solution:

Solution:
From figure equation of bisectors of lines, $xy = 0$ are $y = \pm x$



$\therefore$ Put $y = \pm x$ in the given equation

$my^2 + (1 - m^2)xy - mx^2 = 0$

$\therefore mx^2 \pm (1 - m^2)x^2 - mx^2 = 0$

$\Rightarrow 1 - m^2 = 0 \Rightarrow m = \pm 1$

Question157

If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}, x > 0$ and $y = 3x, x > 0$, then a belong to [2006]

Options:

A. $\left(0, \frac{1}{2}\right)$

B. $(3, \infty)$

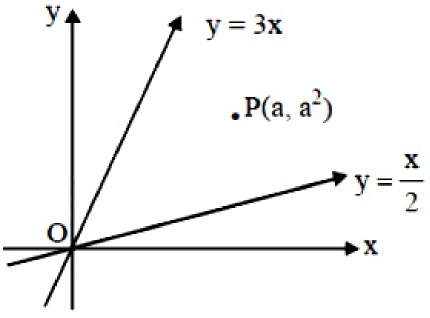
C. $\left(\frac{1}{2}, 3\right)$

D. $\left(-3, -\frac{1}{2}\right)$

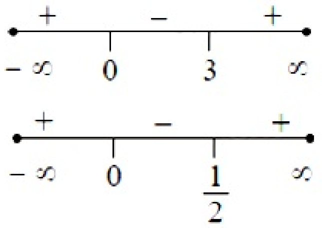
Answer: C

Solution:

Solution:
Clearly for point P,



$a^2 - 3a < 0$ and $a^2 - \frac{a}{2} > 0$



$\Rightarrow \frac{1}{2} < a < 3$

Question158

A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A. Its equation is [2006]

Options:

A. $x + y = 7$

B. $3x - 4y + 7 = 0$

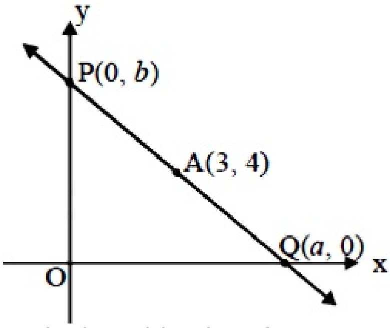
C. $4x + 3y = 24$

D. $3x + 4y = 25$

Answer: C

Solution:

Solution:



∵A is the mid point of PQ

$\therefore \frac{a+0}{2} = 3, \ \frac{0+b}{2} = 4 \Rightarrow a = 6, b = 8$

$\therefore$ Equation of line is $\frac{x}{6} + \frac{y}{8} = 1$

or $4x + 3y = 24$

Question159

If a vertex of a triangle is (1,1) and the mid points of two sides through this vertex are (-1,2) and (3,2) then the centroid of the triangle is [2005]

Options:

A. $\left(-1, \ \frac{7}{3}\right)$

B. $\left(\frac{-1}{3}, \ \frac{7}{3}\right)$

C. $\left(1, \ \frac{7}{3}\right)$

D. $\left(\frac{1}{3}, \ \frac{7}{3}\right)$

Answer: C

Solution:

Solution:
Vertex of triangle is (1,1) and midpoint of sides through this vertex is (-1,2) and (3,2)
Co-ordinate of B is (x, y)
co-ordinates of B is $\left(\frac{1+x}{2} = -1, \ \frac{1+y}{2} = 1\right) : : ((x, y) : (-3, 3))$
Similarly, co-ordinate of C comesout to be (5,3)
Thus centroid is, $\frac{1-3+5}{3}, \ \frac{1+3+3}{3} \Rightarrow \left(1, \ \frac{7}{3}\right)$ … (hint: using formula of centroid)

Question160

The line parallel to the x -axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is [2005]

Options:

A. below the x - axis at a distance of $\frac{3}{2}$ from it

B. below the x - axis at a distance of $\frac{2}{3}$ from it

C. above the x - axis at a distance of $\frac{3}{2}$ from it

D. above the x - axis at a distance of $\frac{2}{3}$ from it

Answer: A

Solution:

Solution:

The eqn. of line passing through the intersection of lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$ is $ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$

$\Rightarrow (a + b\lambda)x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$

Required line is parallel to x -axis.

$\therefore a + b\lambda = 0 \Rightarrow \lambda = -a / b$

$\Rightarrow ax + 2by + 3b - \frac{a}{b}(bx - 2ay - 3a) = 0$

$\Rightarrow ax + 2by + 3b - ax + \frac{2a^2}{b}y + \frac{3a^2}{b} = 0$

$y\left(2b + \frac{2a^2}{b}\right) + 3b + \frac{3a^2}{b} = 0$

$y\left(\frac{2b^2 + 2a^2}{b}\right) = -\left(\frac{3b^2 + 3a^2}{b}\right)$

$y = \frac{-3(a^2 + b^2)}{2(b^2 + a^2)} = \frac{-3}{2}$

So it is 3 / 2 units below x -axis.

Question161

The equation of the straight line passing through the point (4,3) and making intercepts on the co-ordinate axes whose sum is -1 is [2004]

Options:

A. $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

B. $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

C. $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$

D. $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

Answer: A

Solution:

Solution:

Let the required line be $\frac{x}{a} + \frac{y}{b} = 1$ ....(i)
then $a + b = -1 \Rightarrow b = -a - 1$ ....(ii)
(i) passes through (4, 3), $\Rightarrow \frac{4}{a} + \frac{3}{b} = 1$
$\Rightarrow 4b + 3a = ab$ ....(iii)
Putting value of b from (ii) in (iii), we get
$a^2 - 4 = 0 \Rightarrow a = \pm 2 \Rightarrow b = -3$
or 1 ∴ Equations of straight lines are
$\frac{x}{2} + \frac{y}{-3} = 1$ or $\frac{x}{-2} + \frac{y}{1} = 1$

Question162

Let A(2, −3) and B(−2, 3) be vertices of a triangle ABC. If the centroid of this triangle moves on the line 2x + 3y = 1, then the locus of the vertex C is the line [2004]

Options:

A. $3x - 2y = 3$

B. $2x - 3y = 7$

C. $3x + 2y = 5$

D. $2x + 3y = 9$

Answer: D

Solution:

Solution:
Let the vertex C be (h, k), then the
centroid of ΔABC is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$
$= \left(\frac{2 - 2 + h}{3}, \frac{-3 + 1 + k}{3} \right)$
$= \left(\frac{h}{3}, \frac{-2 + k}{3} \right)$ · It lies on $2x + 3y = 1$
$\Rightarrow \frac{2h}{3} - 2 + k = 1 \Rightarrow 2h + 3k = 9$
⇒ Locus of C is $2x + 3y = 9$

Question163

If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals [2004]

Options:

A. -3

B. 1

C. 3

D. 1

Answer: A

Solution:

Solution:
$3x + 4y = 0$ is one of the line of the pair equations. of lines
$6x^2 - xy + 4cy^2 = 0$, Put $y = -\frac{3}{4}x$
we get, $6x^2 + \frac{3}{4}x^2 + 4c\left(-\frac{3}{4}x\right)^2 = 0$
$\Rightarrow 6 + \frac{3}{4} + \frac{9c}{4} = 0 \Rightarrow c = -3$

Question164

If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product c has the value
[2004]

Options:

A. -2

B. -1

C. 2

D. 1

Answer: C

Solution:

Solution:
Let the lines be $y = m_1x$ and $y = m_2x$ then
$m_1 + m_2 = -\frac{2c}{7}$ and $m_1m_2 = -\frac{1}{7}$
Given that $m_1 + m_2 = 4m_1m_2$
$\Rightarrow -\frac{2c}{7} = -\frac{4}{7} \Rightarrow c = 2$

Question165

If the equation of the locus of a point equidistant from the point (a_1, b_1) and (a_2, b_2) is $(a_1 - b_2)x + (a_1 - b_2)y + c = 0$, then the value of $'c'$ is
[2003]

Options:

A. $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

B. $\frac{1}{2}a_2^2 + b_2^2 - a_1^2 - b_1^2$

C. $a_1^2 - a_2^2 + b_1^2 - b_2^2$

D. $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$

Answer: B

Solution:

Solution:
$(x - a_1)^2 + (y - b_1)^2 = (x - a_2)^2 + (y - b_2)^2$
$(a_1 - a_2)x + (b_1 - b_2)y + \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2) = 0$
Comparing with given eqn. we get
$c = \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$

Question166

Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and $(1, 0)$, where t is a parameter, is [2003]

Options:

A. $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

B. $(3x - 1)^2 + (3y)^2 = a^2 - b^2$

C. $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

D. $(3x + 1)^2 + (3y)^2 = a^2 + b^2$

Answer: C

Solution:

Solution:
We know that centroid
$(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$
$x = \frac{a\cos t + b\sin t + 1}{3}$
$\Rightarrow a\cos t + b\sin t = 3x - 1$
$y = \frac{a\sin t - b\cos t}{3}$
$\Rightarrow a\sin t - b\cos t = 3y$
Squaring and adding,
$(3x - 1)^2 + (3y)^2 = a^2 + b^2$

Question167

If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) [2003]

Options:

A. are vertices of a triangle

B. lie on a straight line

C. lie on an ellipse

D. lie on a circle.

Answer: B

Solution:

Solution:
Taking co-ordinates as
$A\left(\frac{x}{r}, \frac{y}{r}\right)$; $B(x, y)$ and $C(xr, yr)$
Then slope of line joining
$A\left(\frac{x}{r}, \frac{y}{r}\right)$, $B(x, y) = \dfrac{y\left(1-\frac{1}{r}\right)}{x\left(1-\frac{1}{r}\right)} = \dfrac{y}{x}$
and slope of line joining $B(x, y)$ and $C(xr, yr)$
$= \dfrac{y(r-1)}{x(r-1)} = \dfrac{y}{x}$
$\therefore m_1 = m_2$
$\because$ Slope of AB and BC are same and one point B common.
$\Rightarrow$ Points lie on the straight line.

Question168

A square of side a lies above the x -axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x -axis. The equation of its diagonal not passing through the origin is [2003]

Options:

A. $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

B. $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

C. $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

D. $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

Answer: A

Solution:

Solution:
Co-ordinates of A = $(a\cos\alpha, a\sin\alpha)$ Equation of OB,

$y = \tan\left(\frac{\pi}{4} + \alpha\right)x$

$CA\perp^{r}$ to OB

$\therefore$ Slope of CA $= -\cot\left(\frac{\pi}{4} + \alpha\right)$

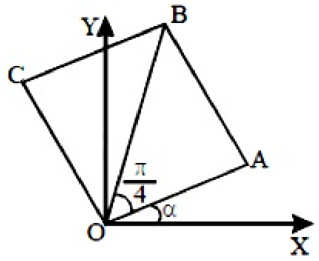
Equation of CA

$y - a\sin\alpha = -\cot\left(\frac{\pi}{4} + \alpha\right)(x - a\cos\alpha)$

$\Rightarrow (y - a\sin\alpha)\left(\tan\left(\frac{\pi}{4} + \alpha\right)\right) = (a\cos\alpha - x)$

$$\Rightarrow (y - a\sin\alpha)\left(\frac{\tan\frac{\pi}{4} + \tan\alpha}{1 - \tan\frac{\pi}{4}\tan\alpha}\right) = (a\cos\alpha - x)$$

$\Rightarrow (y - a\sin\alpha)(1 + \tan\alpha) = (a\cos\alpha - x)(1 - \tan\alpha)$
$\Rightarrow (y - a\sin\alpha)(\cos\alpha + \sin\alpha)$
$= (a\cos\alpha - x)(\cos\alpha - \sin\alpha)$



$\Rightarrow y(\cos + \sin\alpha) - a\sin\alpha\cos\alpha - a\sin^2\alpha$
$= a\cos^2\alpha - a\cos\alpha\sin\alpha - x(\cos\alpha - \sin\alpha)$
$\Rightarrow y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$
$y(\sin\alpha + \cos\alpha) + x(\cos\alpha - \sin\alpha) = a$

Question169

If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then [2003]

Options:

A. $pq = -1$

B. $p = q$

C. $p = -q$

D. $pq = 1$.

Answer: A

Solution:

Solution:
Equation of bisectors of second pair of straight lines is,
$qx^2 + 2xy - qy^2 = 0 \ldots$ (i)
It must be identical to the first pair ....
$x^2 - 2pxy - y^2 = 0 \ldots$ (ii)
from (i) and (ii)

$\frac{q}{1} = \frac{2}{-2p} = \frac{-q}{-1} \Rightarrow pq = -1$

Question170

A triangle with vertices (4,0),(-1,-1),(3,5) is [2002]

Options:

A. isosceles and right angled

B. isosceles but not right angled

C. right angled but not isosceles

D. neither right angled nor isosceles

Answer: A

Solution:

Solution:
$AB = \sqrt{(4+1)^2 + (0+1)^2} = \sqrt{26}$
$BC = \sqrt{(3+1)^2 + (5+1)^2} = \sqrt{52}$
$CA = \sqrt{(4-3)^2 + (0-5)^2} = \sqrt{26}$
$\because AB = CA$
$\therefore$ Isosceles triangle
$\because (\sqrt{26})^2 + (\sqrt{26})^2 = 52$
$BC^2 = AB^2 + AC^2$
$\therefore$ right angled triangle,
So, the given triangle is isosceles right angled.

Question171

Locus of mid point of the portion between the axes of $x\cos\alpha + y\sin\alpha = p$ where p is constant is [2002]

Options:

A. $x^2 + y^2 = \frac{4}{p^2}$

B. $x^2 + y^2 = 4p^2$

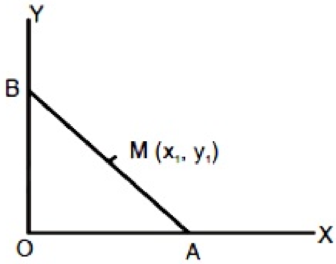
C. $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{p^2}$

D. $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

Answer: D

Solution:

Solution:



Equation of AB is

$x\cos\alpha + y\sin\alpha = p$;

$\Rightarrow \frac{x\cos\alpha}{p} + \frac{y\sin\alpha}{p} = 1$

$\Rightarrow \frac{x}{p / \cos\alpha} + \frac{y}{p / \sin\alpha} = 1$

So, co-ordinates of A and B are

$\left(\frac{p}{\cos\alpha}, 0\right)$ and $\left(0, \frac{p}{\sin\alpha}\right)$

So, coordinates of midpoint of AB are

$M(x_1, y_1) = \left(\frac{p}{2\cos\alpha}, \frac{p}{2\sin\alpha}\right)$

$x_1 = \frac{p}{2\cos\alpha \&} y_1 = \frac{p}{2\sin\alpha}$

$\Rightarrow \cos\alpha = p / 2x_1$ and $\sin\alpha = p / 2y_1$

$\because \cos^2\alpha + \sin^2\alpha = 1$

Locus of (x_1, y_1) is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$.

Question172

The pair of lines represented by $3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ are perpendicular to each other for [2002]

Options:

A. two values of a

B. ∀a

C. for one value of a

D. for no values of a

Answer: A

Solution:

Solution:

We know that pair of straighty lines

$ax^2 + 2hxy + by^2 = 0$ are perpendicular when $a + b = 0$

$3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0$

$\Rightarrow a = \frac{-3 \pm \sqrt{9 + 8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$

Question173

If the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersect on the y -axis then
[2002]

Options:

A. $2fgh = bg^2 + ch^2$

B. $bg^2 \neq ch^2$

C. $abc = 2fgh$

D. none of these

Answer: A

Solution:

Solution:
Put $x = 0$ in the given equation
$\Rightarrow by^2 + 2fy + c = 0$
For unique point of intersection, $f^2 - bc = 0$
$\Rightarrow af^2 - abc = 0$
We know that for pair of straight line
$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
$\Rightarrow 2fgh - bg^2 - ch^2 = 0$
